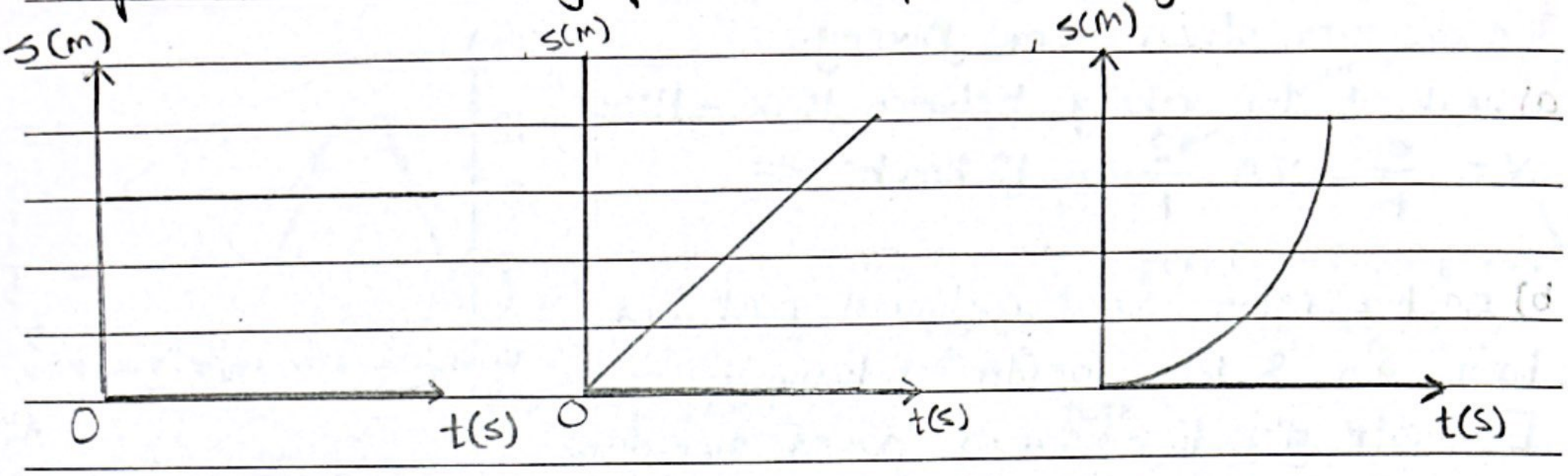
Ch 2 Kinematics

Displacement - time graphs -> slope = velocity

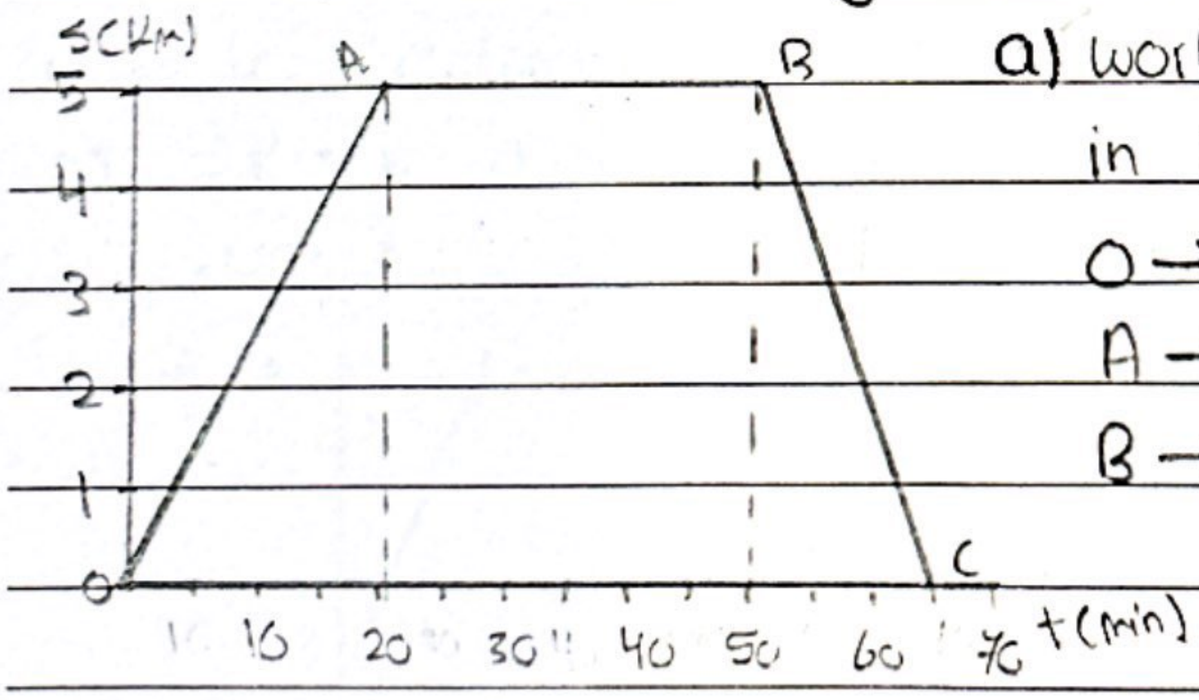


Stationary | constant velocity | increasing velocity

- Average velocity = displacement from origin / time taken
- Average speed = Total distance travelled / time taken

Example ①

A cyclist rides in a straight line for 20 mins. She waits for ½ hr, then returns in a straight line to starting point in 15 mins.



a) workout the av. velocity in each stage in kmh^-

$O \to A$ AV. $V = \frac{5}{20 \div 60} = 15\ kmh^-$ ⇐

$A \to B$ AV. $V = 0\ kmh^-$ ⇐

$B \to C$ AV. $V = \frac{-5}{15 \div 60} = -20\ kmh^-$

b) average velocity of the whole journey

$= \frac{+5 - 5}{65 \div 60} = 0\ kmh^-$

N.B. When somthing returns to start point then total displacement = 0

c) average speed for the whole journey

$s = \frac{5+5}{65 \div 60}$ $s = 9.23\ kmh^-$ 3sf. ⇐

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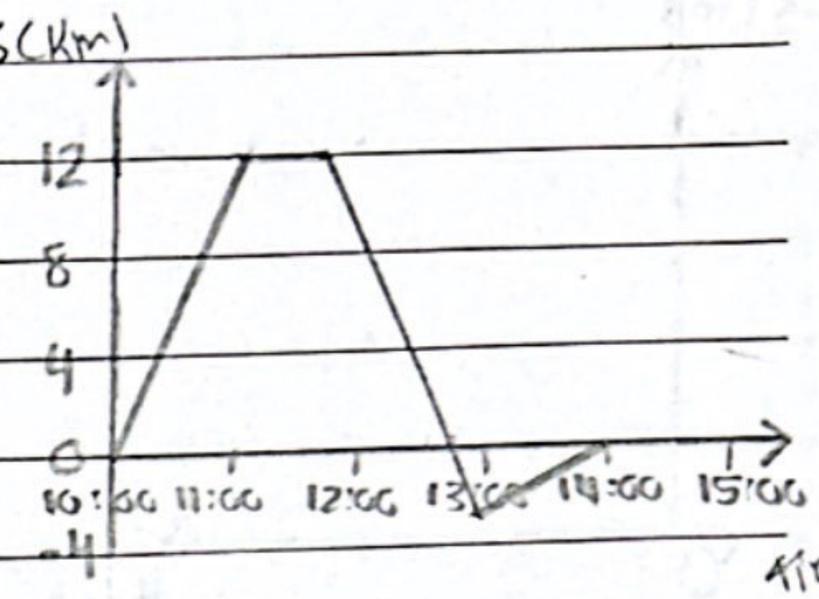
Example ②

Sarah left home at 10:00 and cycled North in a straight line. The diagram shows her journey.



a) workout the velocity between 10:00 - 11:00

$v = \frac{s}{t}$ $v = \frac{12}{1} = 12\ kmh^{-}$ ⇐

b) on her return Sarah continued past her home for 3 km before returning.

Estimate the time that Sarah passed her home.

velocity on returning $= \frac{-12+3}{1.5h} = -10\ kmh^{-}$

$time = \frac{|s|}{v} = \frac{3}{10} = 0.3\ hrs$

13:00 - 0.3 hrs = 12:42 ⇐

d) Calculate her average sp for entire journey

av-sp = Total Distance / total time

$= \frac{12 + 15 + 3}{4} = 7.5\ kmh^{-}$ ⇐

c) Work out her velocity for last 2 stages

$v = s/t$

$= -15/1.5$

$= -10\ kmh^{-}$ → before last stage

$v = s/t$

$= 3/1 = 3\ kmh^{-}$ → last stage

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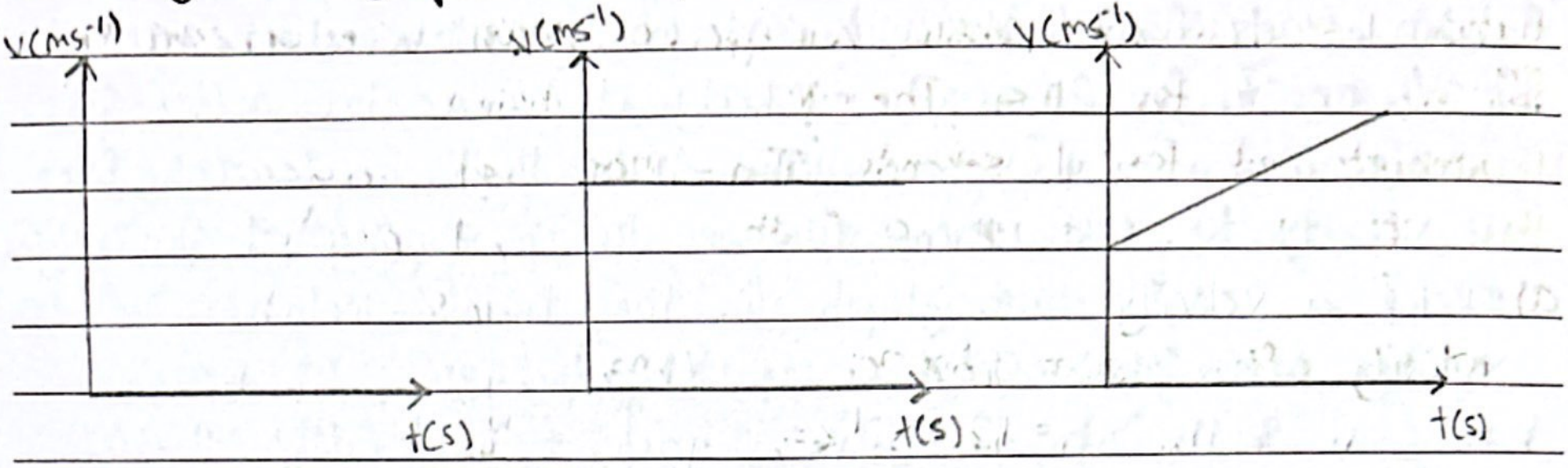
Velocity-time graphs → slope = acceleration



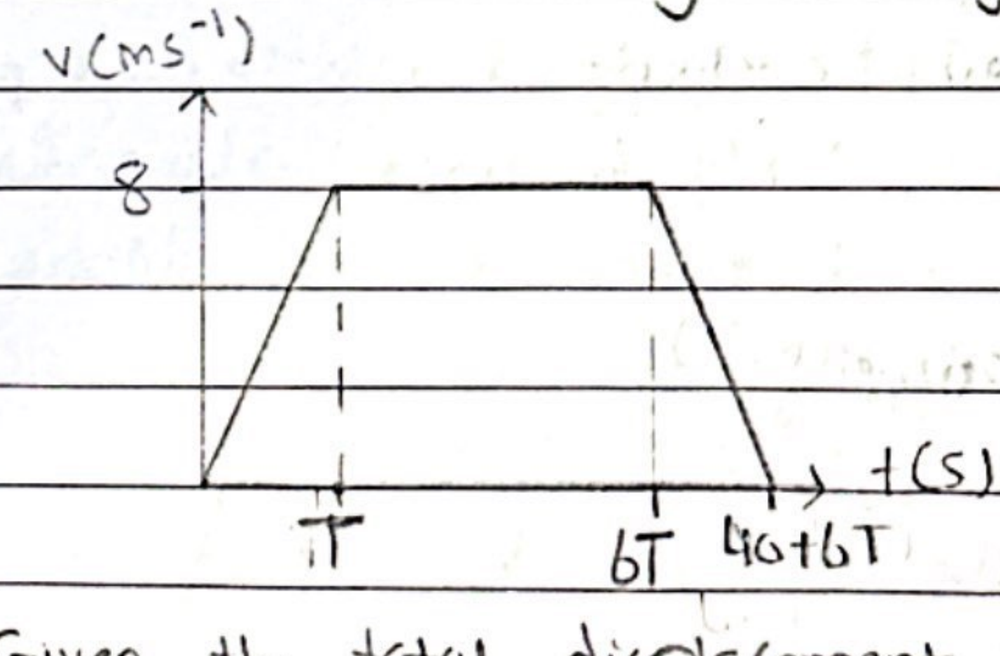
Stationary | constant velocity, acc = zero | increasing velocity, acc = constant

- Area under graph = displacement
- gradient = acceleration

Example ③

A particle moves along a straight line. The particle accelerates uniformly from rest to $8ms^{-1}$ in T seconds. The particle then travels at a constant velocity of $8ms^{-1}$ for 5T seconds. The particle decelerates uniformly to rest in a further 40 seconds.

a) Sketch a velocity-time graph



b) Given the total displacement is 600m find T

$600 = \frac{1}{2}(5T + 40 + 6T)8$

$600 = 4(11T + 40)$

$600 = 44T + 160$

$440 = 44T$

$T = 10s \Leftarrow$

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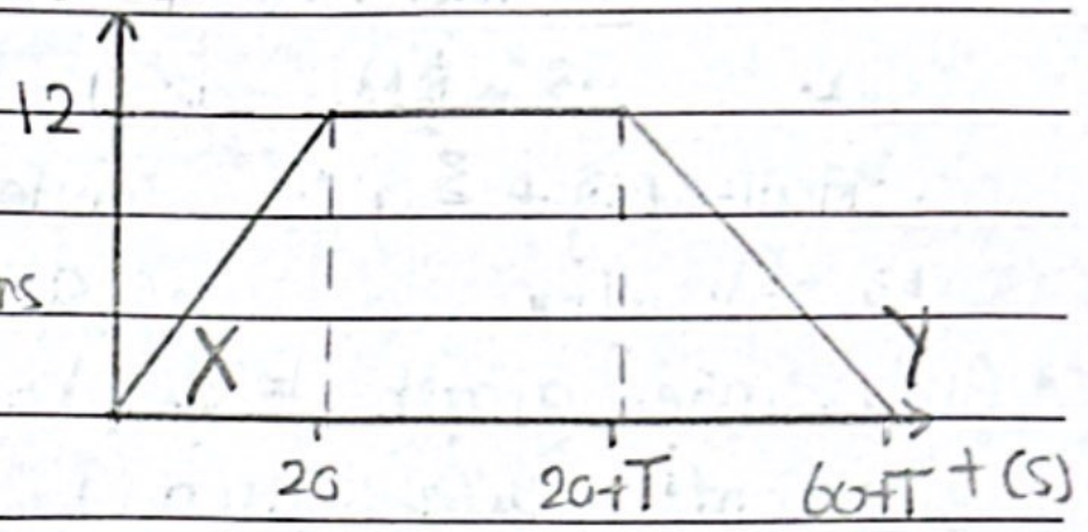
Example ④

A train starts from a station X and moves with constant acceleration of $0.6\ ms^{-2}$ for 20 s. The velocity it has reached after 20 s is maintained for T seconds. The train then decelerates from this velocity to rest in a further 40 s, stopping at station Y.

a) Sketch a velocity-time graph for the trains motion

velocity after 20 s $= 0.6 \times 20$

$= 12\ ms^{-1} \Leftarrow$



b) given distance between the stations is 4.2 Km, find T

$4200 = \frac{1}{2}(T+60+T)\,12$

$4200 = 6(2T+60)$

$700 = 2T+60$

$640 = 2T$

$T = 320 \Leftarrow$

c) distance travelled when at constant velocity

$D = s \times t$

$= 12 \times 320$

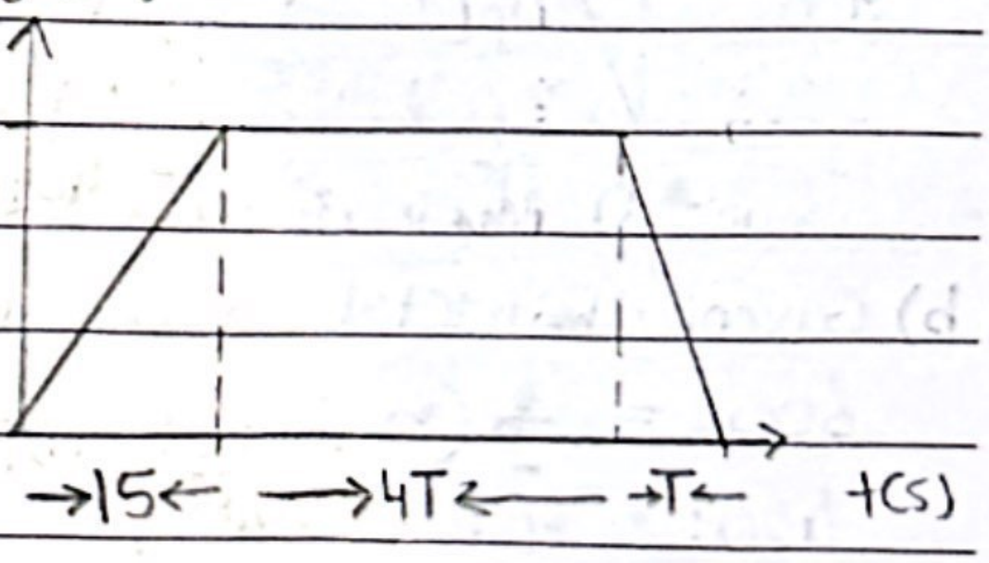
$= 3840\ m \Leftarrow$

Example ⑤

A particle moves along a straight. The particle accelerates from rest to $10ms^{-1}$ in 15 s. The particle then moves at constant velocity of $10ms^{-1}$ for a period of time. The particle then decelerates uniformly to rest. The time for which particle's speed is constant is 4 times when its decelerating.

a) Sketch a velocity-time graph.

b) given the total displacement = 480m find the total time for which the particle travels.



$480 = \frac{1}{2}(4T+5T+15)\,10$

$480 = 5(9T+15)$

$480 = 45T + 75$

$405 = 45T$

$T = 9 \Leftarrow$

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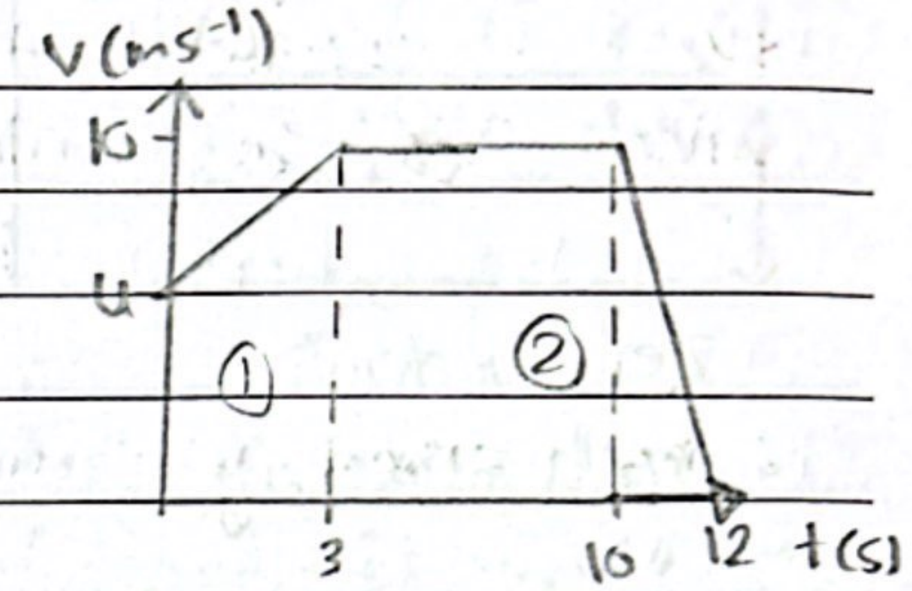
Example ⑥

A particle moves 100m is a straight line. The diagram shows its motion. The particle starts at $u\ ms^{-1}$ and accelerates to velocity of $10\ ms^{-1}$ in 3s. The velocity of $10\ ms^{-1}$ is maintained for 7s and then the particle decelerates to rest in a further 2s.



a) find u

$$100\,m = \frac{1}{2}(10+u)3 + \frac{1}{2}(9+7)10$$

$$100 = 15 + \frac{3}{2}u + 80$$

$$20 = 15 + \frac{3}{2}u$$

$$5 = \frac{3}{2}u$$

$$u = \frac{5 \cdot 2}{3} \qquad u = \frac{10}{3} \Leftarrow$$

b) acceleration of particle in first 3 seconds

$$a = \frac{v-u}{t} = \frac{10 - \frac{10}{3}}{3} = 2.22\ ms^{-2} \Leftarrow$$

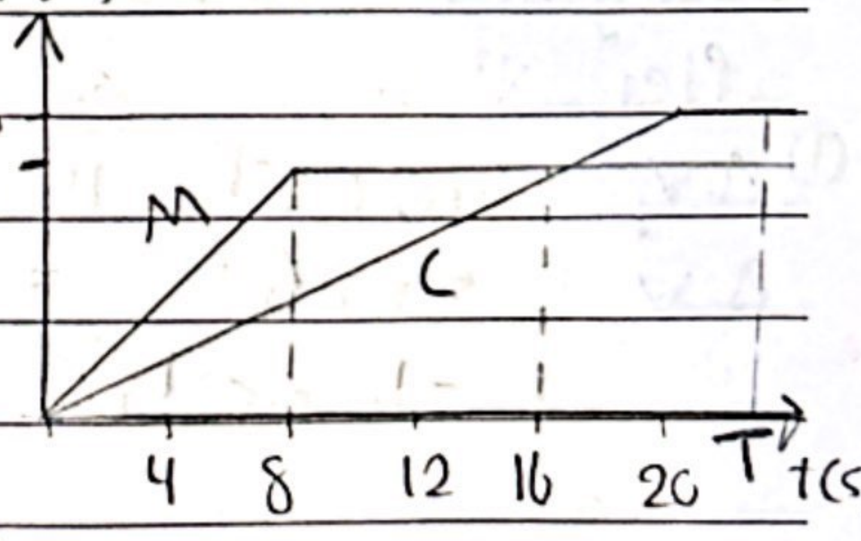
very imp.

Example ⑦

A motorcyclist M leaves a road junction at $t = 0$. She accelerates from rest at a rate $3ms^{-2}$ for 8 seconds and maintains the speed she has reached. A car C leaves the same junction at $t = 0$. The car accelerates from rest to $30\ ms^{-1}$ in 20 s and then maintains the velocity of $30\ ms^{-1}$. C passes M as they both pass a pedestrian

a) on the same diagram sketch a speed-time graph for M and C

$3 \times 8 = 24\ ms^{-1} \rightarrow$ sp M



b) Find distance of the pedestrian from road junction.

① Let T be the time at which C overtakes M

distance M = distance C

$$\frac{1}{2}(T-8+T)24 = \frac{1}{2}(T-20+T)30$$

$$24T - 96 = 30T - 300$$

$$6T = 204$$

$$T = 34$$

② $d = \frac{1}{2}(2(34)-8) \times 24$

$d = 720\ m \Leftarrow$

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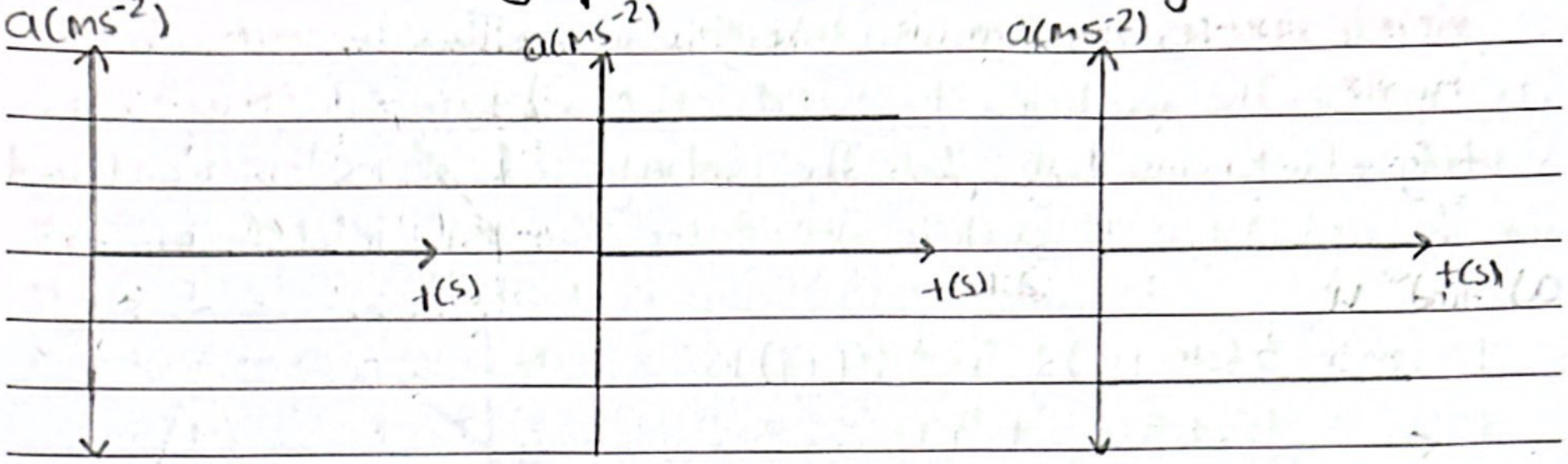
Acceleration-time graphs → Area = Δ velocity



Zero acceleration	+ve constant acc	-ve constant acc
constant velocity	increasing velocity	decreasing velocity

- Area under graph = change in velocity in that time interval

N.B.

-ve acceleration = particle slowing down

-ve velocity = particle travelling backwards

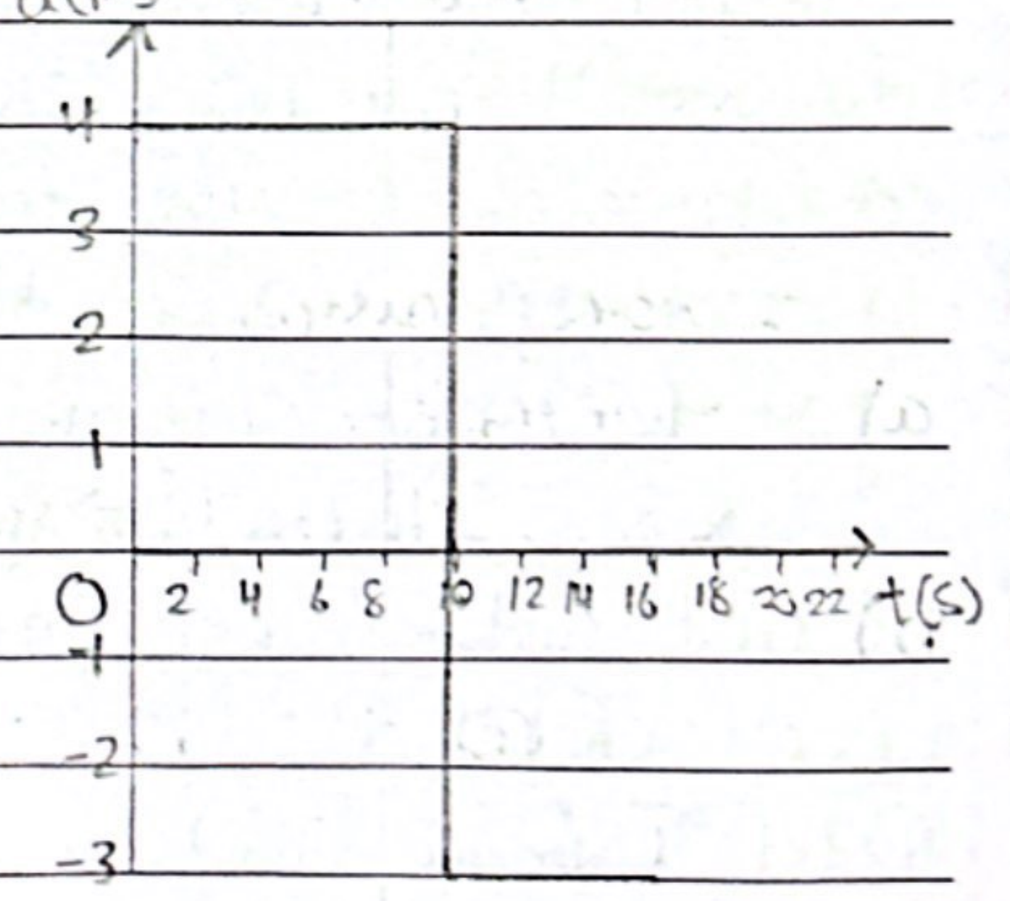
Example ⑧

The acceleration-time graph shows the motion of a car over a 20 second period.

a) calculate the change in velocity between $t = 6$ and $t = 10$ s.

$10 \times 4 = 40\ ms^{-1}$

b) Given that the car was initially moving at $15\ ms^{-1}$, calculate velocity after 16 s.



② $\Delta V_1 = 40\ ms^{-1}$ $15 + 40 - 18$

$\Delta V_2 = -3 \times 6 = 37\ ms^{-1}$ ⇐

$= -18\ ms^{-1}$

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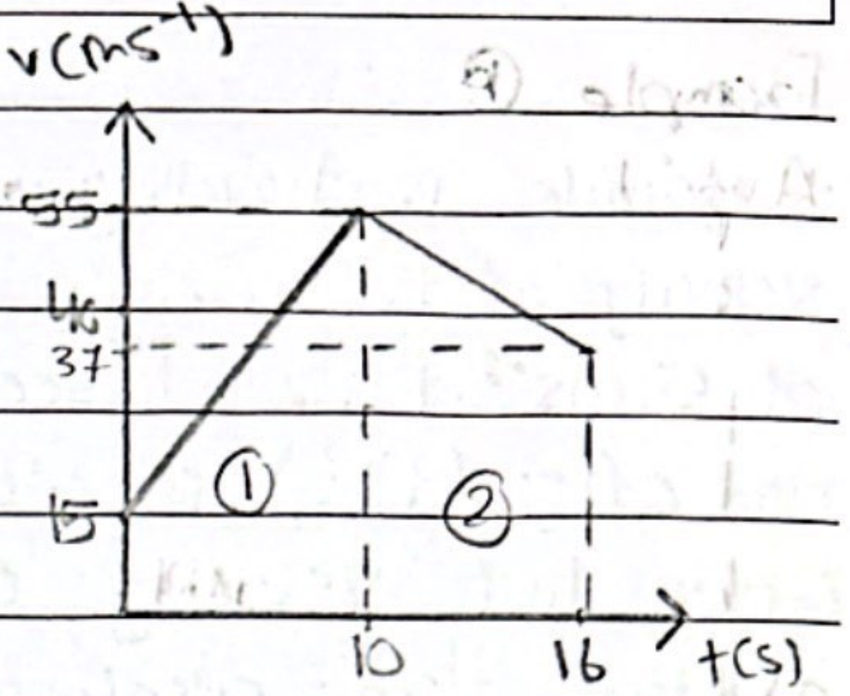
c) Draw a velocity-time graph



d) Find distance travelled in these 16s

$A_1 + A_2$

$\frac{1}{2}(55+15)10 + \frac{1}{2}(37+55)6$

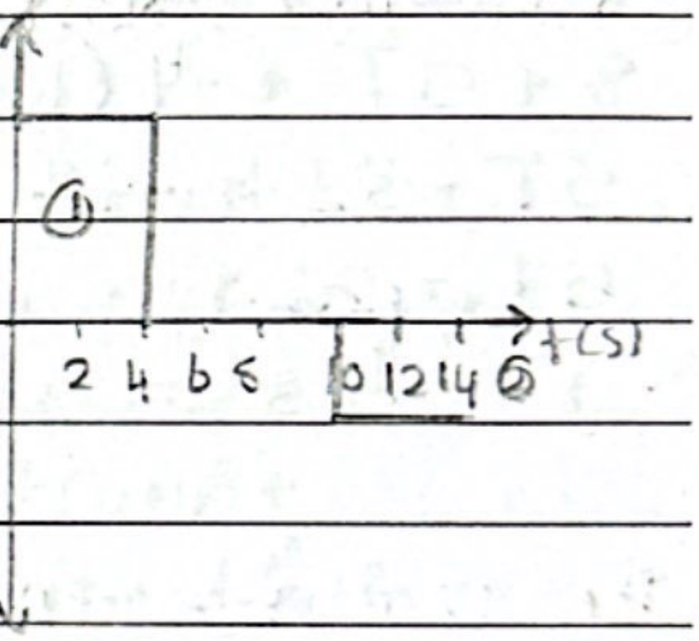
$= 350 + 276$

$= 626\ m \Leftarrow$

Example ⑨

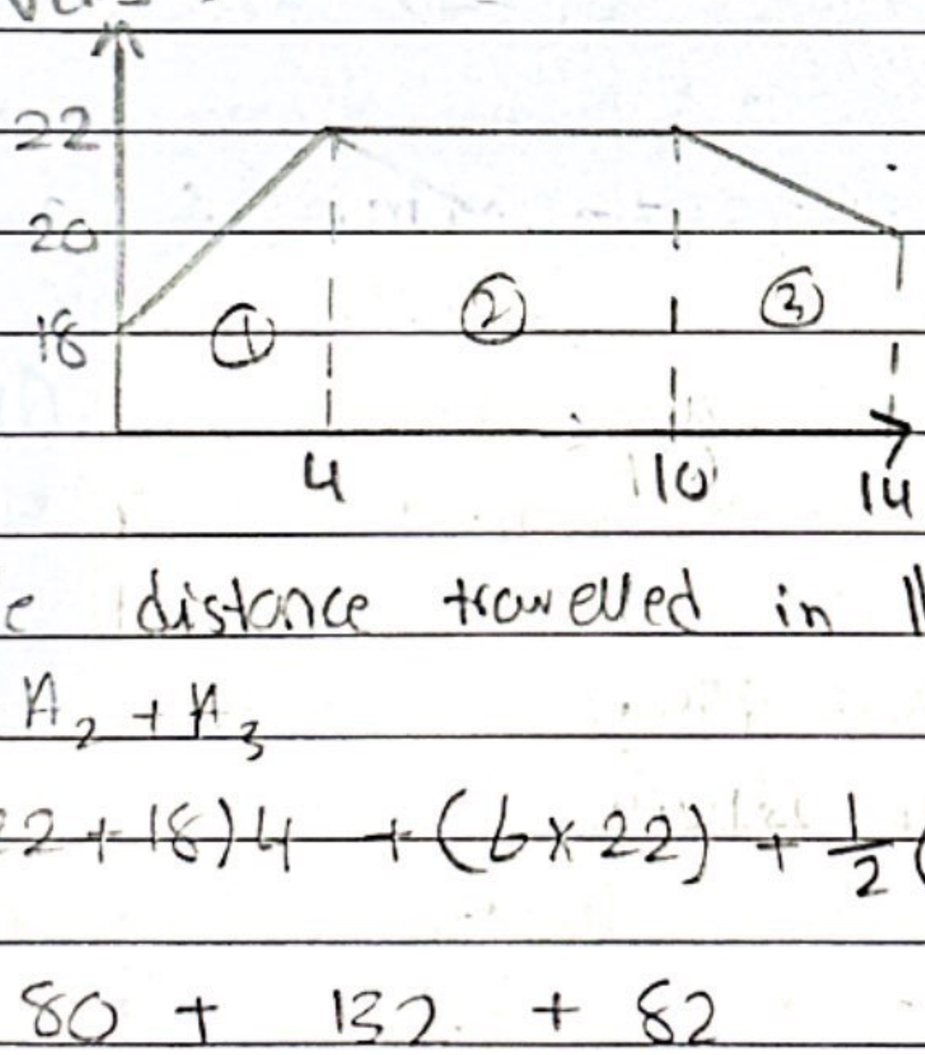
A motorcycle accelerates at a rate of $1ms^{-1}$ for 4 seconds, before travelling at a constant velocity for 6 seconds. It then decelerates at a rate of $0.5\ ms^{-2}$ for 4 seconds.

a) draw an acceleration-time graph



b) Given that initial velocity = $18ms^{-}$

Draw a velocity-time graph



c) calculate distance travelled in 14 seconds

$A_1 + A_2 + A_3$

$= \frac{1}{2}(22+18)4 + (6\times22) + \frac{1}{2}(20+22)4$

$= 80 + 132 + 82$

$= 294\ m \Leftarrow$

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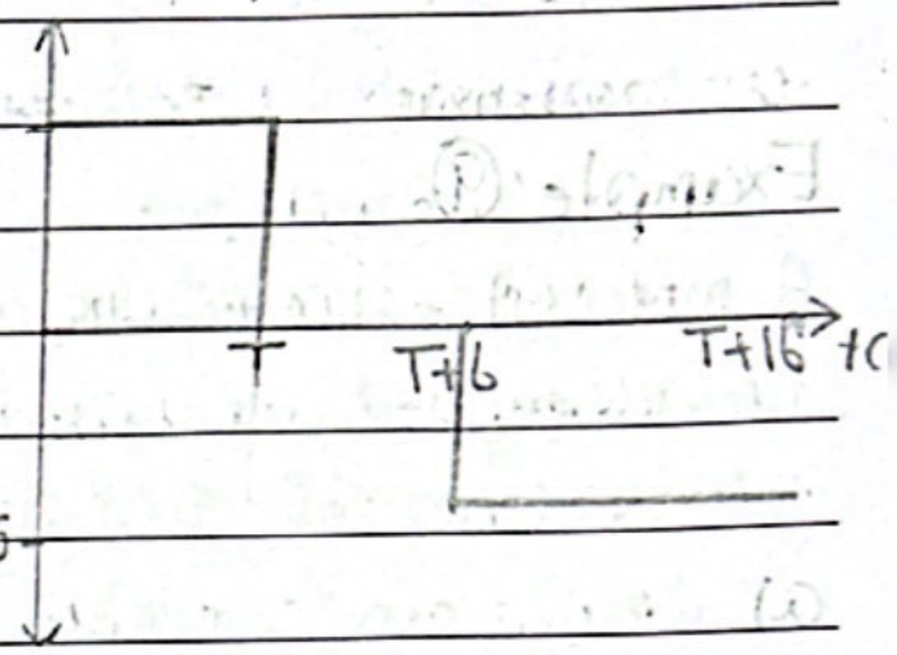
Example ⑯

A particle is travelling along the x-axis At time $t=0$, the velocity of the particle is $8\ ms^{-1}$, and the particle is accelerating at $5\ ms^{-2}$. After T seconds, the particle has constant velocity, and after $(T+6)$ seconds the particle accelerates at $-4\ ms^{-2}$. Given that velocity after $(T+16)$ seconds is $-12\ ms^{-1}$

a) Draw an acceleration-time graph

b) find the value of T



$8\ ms^{-1} = u$

$-12\ ms^{-1} = v$

$8 + (5 \cdot T) + (-4(T+16-(T+6))) = -12$

$8 + 5T + -4(10) = -12$

$5T - 32 = -12$

$5T = 20$

$T = 4\ s \Leftarrow$

SbC

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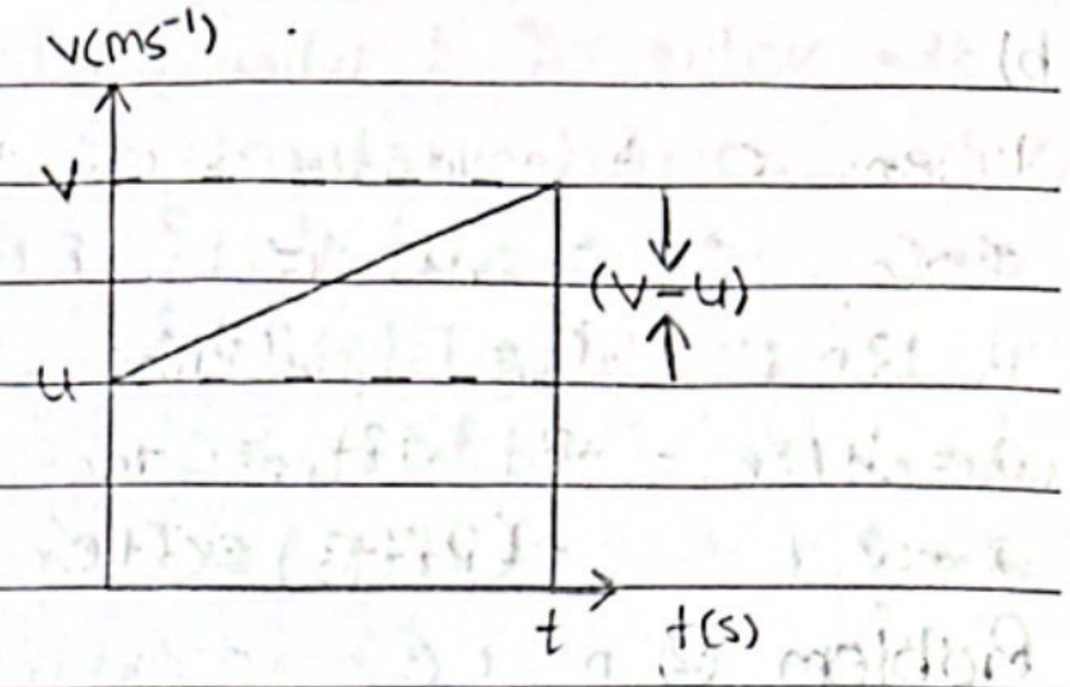
SUVAT formulae

s → displacement
u → initial velocity
v → final velocity
a → acceleration
t → time



① a = gradient = $\frac{v-u}{t}$

rearranged as

$v = u + at$ ⇐

② s = Area under graph

$= \frac{1}{2}(u+v)t$ ⇐

③ substitute $t = \frac{v-u}{a}$ into

$\frac{1}{2}(v+u)t$

$= v^2 = u^2 + 2as$ ⇐

④ substitute $v = u+at$ into

$\frac{1}{2}(v+u)t$

$= s = ut + \frac{1}{2}at^2$ ⇐

⑤ substitute $u = v-at$ into

$s = ut + \frac{1}{2}at^2$

$= s = vt - \frac{1}{2}at^2$ ⇐

SUVAT Equations :-

① $v = u + at$

② $v^2 = u^2 + 2as$

③ $s = \frac{1}{2}(u+v)t$

④ $s = ut + \frac{1}{2}at^2$

⑤ $s = vt - \frac{1}{2}at^2$

problem ①

A particle is moving in a straight horizontal line with constant deceleration $4ms^{-2}$. At time $t=0$ the particle passes through a point O with velocity $13\ ms^{-1}$ travelling towards point A, where OA = 20m Find :

a) the times when the particle passes through A

$a = -4$ $\quad s = ut + \frac{1}{2}at^2$

$u = 13$ $\quad 20 = 13t - 2t^2$

$s = 20$ $\quad 2t^2 - 13t + 20 = 0$

$t = 4, \ t = \frac{5}{2}$ ⇐

N.B.

How ? the particle continues decelerating until it reaches a negative velocity. meaning it will change its direction, coming back to A.

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b) the value of t when particle returns to O

when particle returns to origin, displacement = zero

$s-0$ $\quad s = ut + \frac{1}{2}at^2$

$u=13$ $\quad 0 = 13t - 2t^2$ → $t=0$ X reject

$a=-4$ $\quad 2t^2 - 13t = 0$ → $t = 6.5$ ⇐

$t = ?$ $\quad t(2t-13) = 0$

Problem ②

A particle P is moving on x-axis with constant deceleration $2.5 ms^{-2}$. At time $t=0$, the particle passes through the origin O, moving in the positive direction of x with velocity $15 ms^{-1}$. Find:

a) the time between the instant when P first passes through O and the instant when it returns to O.

O ——— A $\quad a = -2.5 ms^{-2}$

$t=0$ $\quad s = 0$ → as it returned to origin

$s = ut + \frac{1}{2}at^2$ ① $u = 15 ms^{-1}$

$0 = 15t + \frac{1}{2}(-\frac{5}{2})(t)^2$ ②

$0 = 15t - \frac{5}{4}t^2$ ③

$\Rightarrow t = 12$, $t = 0$ X reject ④

Plan (Part b)

get distance OA the multiply by two.

$v^2 = u^2 + 2as$

$0 = 225 + 2(-2.5)s$

$0 = 225 - 5s$

$5s = 225$

$s = 45 m$ ⇐ → $45 \times 2 = \boxed{90 m}$

b) the total distance travelled by P during this time. (N.B.)

$a = -2.5 ms^{-2}$

$u = 15 ms^{-1}$

$d = ?$

$v = 0 ms^{-1}$

→ why? as P reaches point A and changes direction it encounters a turning point, where $v = 0 ms^{-1}$ (maximum point)

IMPORTANT !!

O → A → at a turning point, velocity = zero !!

O ←

When particle moves from O to A then back to O, the displacement is ZERO !!

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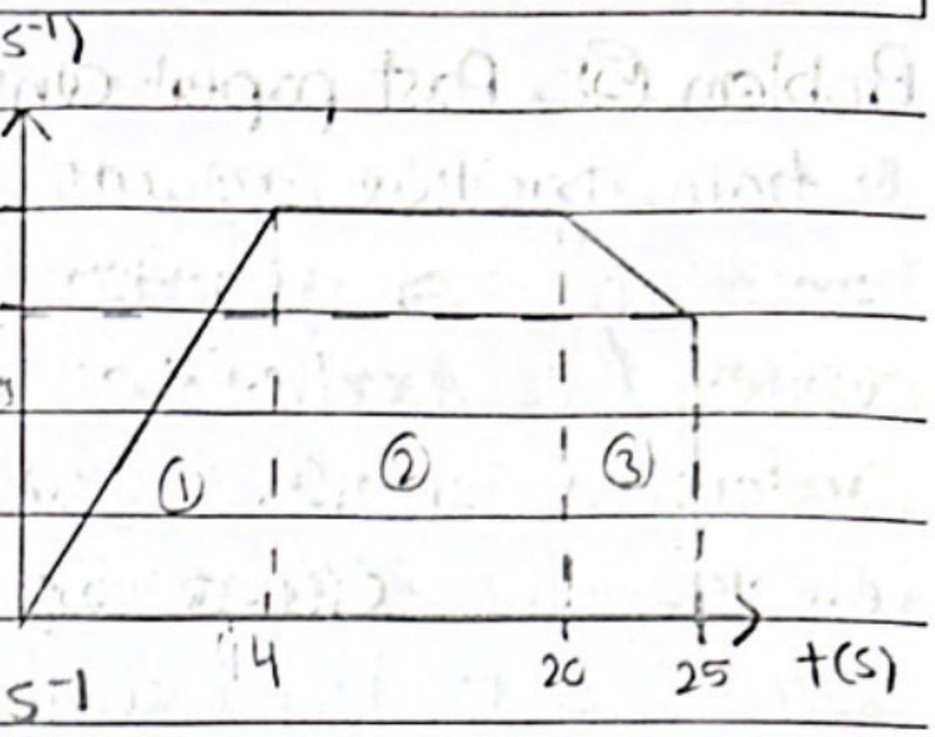
Problem ③

A sprinter runs a race of 200m. Her Total time for running the race is 25 s. She starts from rest and accelerates uniformly to $9ms^{-1}$ in 4 seconds. The sp of $9 ms^{-1}$ is maintained for 16s and then she decelerates uniformly to $u ms^{-1}$ at the end of the race. Calculate



a) distance covered in the first 20 s

$s = \frac{1}{2}(16 + 20)9$

$s = 162 m \Leftarrow$

b) the value of u

using section ③ → 200 − 162

$= 38m$

$38 = \frac{1}{2}(u + 9)5$

$15.2 = u + 9$

$u = 6.2 ms^{-1} \Leftarrow$

c) the deceleration in last 5 s

$a = \frac{v-u}{t} = \frac{6.2-9}{5}$

$a = -0.56$

$dec = 0.56 ms^{-2} \Leftarrow$

Problem ④

A particle is moving along the x-axis with constant deceleration $5ms^{-2}$. At time $t = 0$, the particle passes through the origin O with velocity $12 ms^{-1}$ in the positive direction. At time t seconds, the particle passes through the point A with x coordinates 8. Find :-

a) the values of t

$s = 8$

$a = -5$

$u = 12$

$s = ut + \frac{1}{2}at^2$

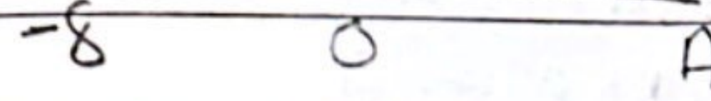
$8 = 12t + \frac{1}{2}(-5)t^2$

$8 = 12t - \frac{5}{2}t^2$

$\frac{5}{2}t^2 - 12t + 8 = 0$

$t = 0.8, \quad t = 4 \Leftarrow$

b) the speed as it passes the x-coordinate −8.



$s = -8$

$u = 12$

$a = -5$

$v^2 = u^2 + 2as$

$v^2 = 144 + 80$

$v^2 = 224$

$v = \pm 15$

$sp = 15 ms^{-1}$ ↗

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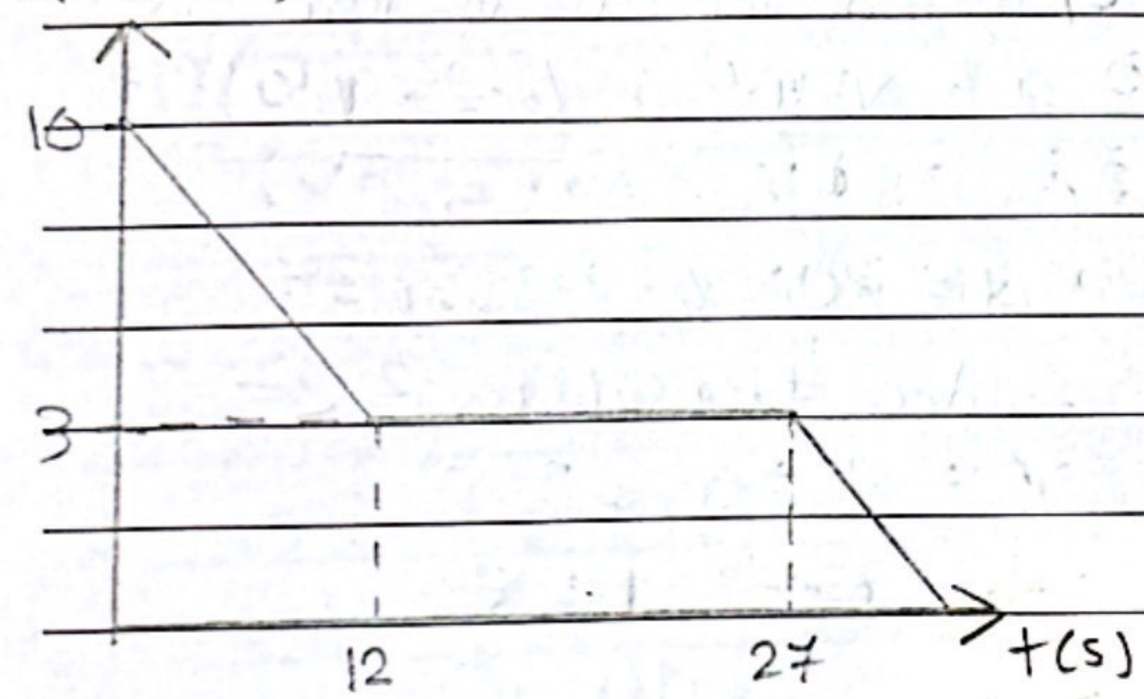
Problem ⑤, Past paper Qn (IMP)

A train is travelling at 10 ms^{-1} on a straight horizontal track. The driver sees a red signal 135m ahead and immediately applies the brakes. The train immediately decelerates with constant deceleration for 12s, reducing its speed to 3 ms^{-1}. The driver releases the brakes allowing the train to travel at a constant speed 3 ms^{-1} for a further 15s. He then applies the brakes again and the train slows down with constant deceleration, coming to a rest as it reaches the signal.

a) Sketch a speed-time graph to show the train's motion



b) find distance between the moment of braking to the moment the train reaches 3 ms^{-1}.

$u = 10 ms^{-1}$
$v = 3 ms^{-1}$
$t = 12$

$s = \frac{1}{2}(u+v)t$
$s = \frac{1}{2}(10+3)12$
$s = 78\ m$ ⇐

c) Find the total time from the moment when the brakes are first applied to the moment when the train comes to rest.

① Total distance = 135
$d_1 = 78m$
$d_2 = 15 \times 3$
$= 45m$
$d_3 = 135 - (78+45)$
$d_3 = 12m$

② $s = 12$
$v = 0$
$u = 3$

$12 = \frac{1}{2}(0+3)t$
$24 = 3t$
$t = 8s$

③ $27 + 8$
$= 35$ seconds ⇐

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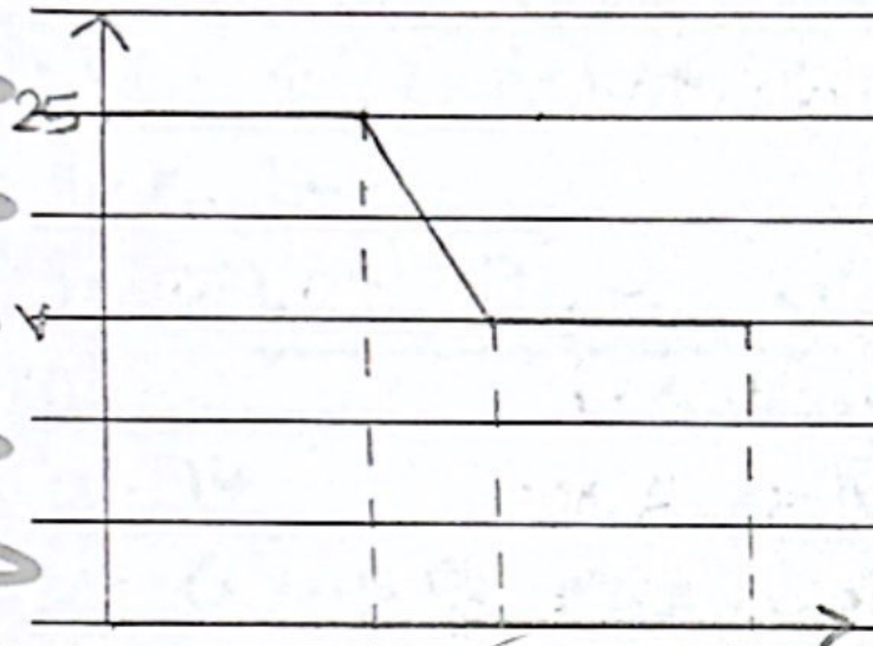
Past paper problem, Example ⑥

A car is moving along a straight road. At time $t=0$ the car passes point A with speed $25\ ms^{-1}$. The car moves with constant speed of $25\ ms^{-1}$ until $t=10s$. The car decelerates uniformly for 8s. At time $t=18s$, the speed of the car is $v\ ms^{-1}$, and this speed is maintained until it reaches point B at $t=30s$.

a) Sketch speed time graph to show motion of the car



b) Given that AB = 526m find value of v

$526 = 30v + \frac{1}{2}(18+10)(25-v)$

$526 = 30v + 14(25-v)$

$526 = 30v - 14v + 350$

$176 = 16v$

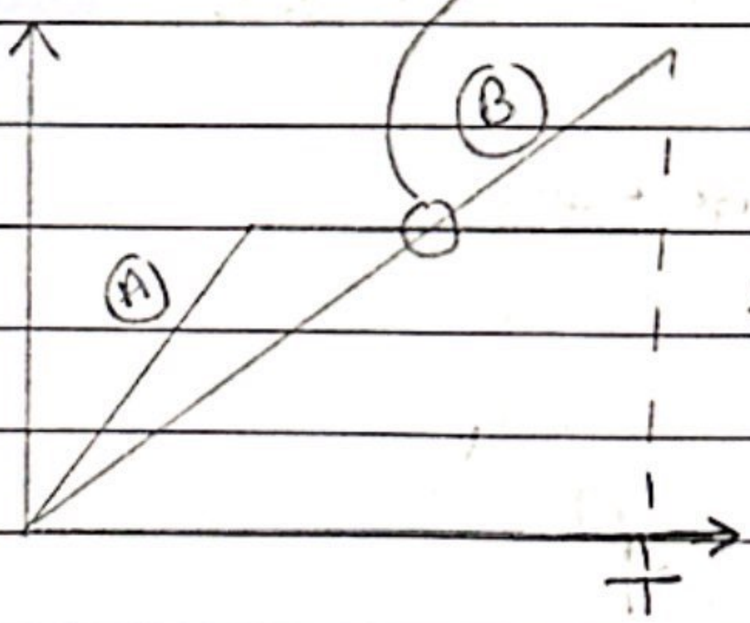
$v = 11\ ms^{-1}$ ⇐

c) find the deceleration between $t=10$ and $t=18s$

$a = \frac{v-u}{t} = \frac{11-25}{8} = -1.75$

deceleration $= 1.75\ ms^{-2}$ ⇐

Overtaking graphs → *Point of intersection is NOT the point where overtaking occurs. However, it is the point where the speed is equal.



* overtaking occurs at time = T where, distance A = distance B

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Important textbook ideas :-

1- A particle P is moving on the x-axis with constant deceleration of $4\ ms^{-2}$. At time $t=0$, P passes through origin O with speed $14\ ms^{-1}$ in the positive direction. The point A lies on the axis and $OA = 22.5\ m$. Find

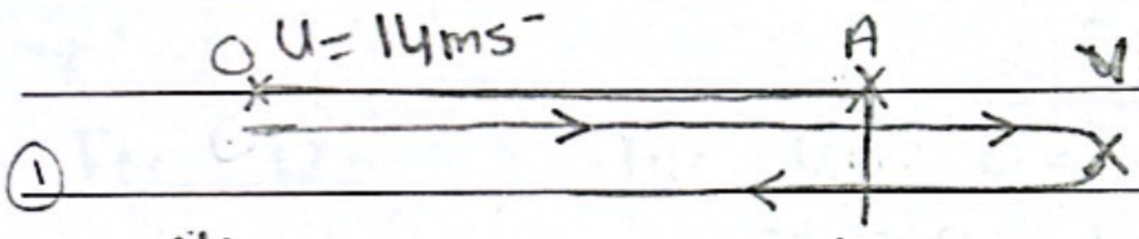
a) the difference between the times when P passes through A

$s = 22.5\ m$ $\quad s = ut + \frac{1}{2}at^2$ $\quad \rightarrow t = 2.5, t = 4.5$

$u = 14\ ms^{-1}$ $\quad 22.5 = 14t - .2t^2$ $\quad \Delta t = 4.5 - 2.5$

$a = -4\ ms^{-2}$ $\quad 2t^2 - 14t + 22.5 = 0$ $\quad \Delta t = 2\ s \Longleftarrow$

b) the total distance travelled by P during the interval between these times.



$u = 14$

$v = 0 \rightarrow$ At max point

$s = ?$

$a = -4\ ms^{-2}$

③ distance from O to max

$= 24.5$

distance from A to max

$= 24.5 - 22.5$

$= 2\ m$

$v^2 = u^2 + 2as$

$0 = 196 + 2(-4)s$

$0 = 196 - 8s$

$8s = 196$

$s = 24.5$

④ $A \rightarrow max = 2m$

$max \rightarrow A = 2m$

$2 + 2 = 4$

distance $= 4\ m \Longleftarrow$

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2- A car is travelling along a straight horizontal road with constant acceleration. The car passes over 3 consecutive points A, B and C where AB = 100m and BC = 300m. The sp of the car at B is 14 ms^{-1} and at C is 20ms^{-1} Find:

a) the acceleration of the car

$u = 14$ $\quad v^2 = u^2 + 2as$

$v = 20$ $\quad 400 = 196 + 600a$

$s = 300$ $\quad 204 = 600a$

$a = 0.34 \Leftarrow$

b) time taken to travel from A to C

$s = 400\,m$ $\quad s = vt + \frac{1}{2}at^2$

$u = ?$ $\quad 400 = 20t - 0.17t^2$

$v = 20$ $\quad 0.17t^2 - 20t + 400 = 0$

$a = 0.34$ $\quad t = 92.1$, $t = 25.5 \Leftarrow$

reject ↙

as at $t = 92.1s$ the car would be moving backwards

→ Why ? due to having -ve velocity- $0.34 = \frac{20 - u}{92.1}$

$u = -11.3$

SbC

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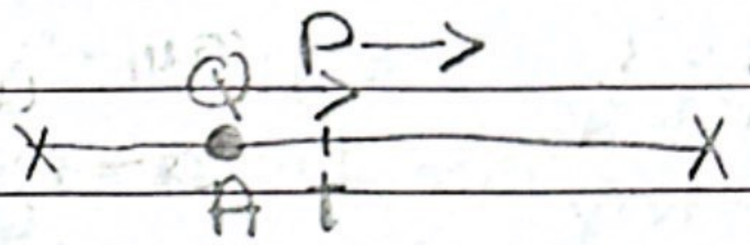
3- Two particles P and Q are moving along the same straight line with constant accelerations $2ms^{-2}$ and $3.6ms^{-2}$ respectively. At $t=0$, P passes through a point A with speed $4ms^{-1}$, one second later Q passes through A with sp $3ms^{-1}$, moving in the same direction as P.

a) Write down expressions for the displacements of p and Q from A, in terms of t, where t seconds is the the time after P pasied through A.



* Since Q is 1 second later than P. this means that at $t=t$, where P is at t, Q is at 1second before t.

$s = ut + \frac{1}{2}at^2$

For P, For Q

$a = 2$ $a = 3.6$

$u = 4$ $u = 3$

time $= t$ time $= (t-1)$

$s_P = 4t + t^2$ ⇐

$s_Q = 3(t-1) + \frac{1}{2}(3.6)(t-1)^2$

$s_Q = 3t - 3 + 1.8(t^2 - 2t + 1)$

$s_Q = 3t - 3 + 1.8t^2 - 3.6t + 1.8$

$s_Q = 1.8t^2 - 0.6t - 1.2$ ⇐

b) Find the value of t when particles meet.

$s_P = s_Q$

$4t + t^2 = 1.8t^2 - 0.6t - 1.2$

$0.8t^2 - 4.6t - 1.2 = 0 \quad \times 5$

$4t^2 - 23t - 6 = 0$

$t = -0.25$ X, $t = 6s$ ⇐

c) Find the distance from A where particls meet by substituting of $t=6$ in either s_P or s_Q

$4(6) + 6^2$

$= 24 + 36$

$= 60m$ ⇐

N.B: When a particle moves after another with 3 seconds time for the latter particle is $(t-3)$

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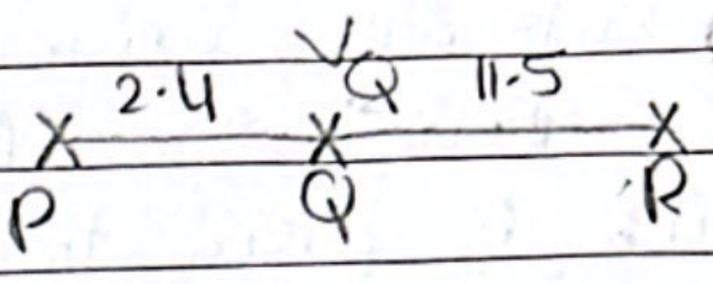
4- In an orienteering competition, a competitor moves in a straight line past three checkpoints P, Q, R, where PQ = 2.4 Km and QR = 11.5 Km. The competitor is a particle moving with constant acceleration. She takes 1 hour to travel from P to Q and 1.5 hours to travel from Q to R Find:-

a) the acceleration of the competitor



let V at Q = V_Q

$s_{P\text{ to }Q} = 2.4$ $s_{Q\text{ to }R} = 11.5$

$t_{P\text{ to }Q} = 1$ $t_{Q\text{ to }R} = 1.5$

①

$s = vt - \frac{1}{2}at^2$ → used from P to Q used from Q to R

$2.4 = V_Q \times 1 - \frac{1}{2}a \times 1$

$2.4 = V_Q - \frac{1}{2}a$

$V_Q = 2.4 + \frac{1}{2}a$

② $s = ut + \frac{1}{2}at^2$

$11.5 = V_Q \times 1.5 + \frac{1}{2} \times a \times 2.25$

$11.5 = \frac{3}{2}V_Q + \frac{9}{8}a$

③ equate to solve for a

$2.4 + \frac{1}{2}a = \frac{23}{3} - \frac{3}{4}a$

$V_Q = (11.5 - 1.125a)\frac{2}{3}$

$V_Q = \frac{23}{3} - \frac{3}{4}a$

$1.25a = \frac{23}{3} - 2.4$

$1.25a = 79/15$

$a = 4.21\ Kmh^{-2}$ →3sf.

$4.21 \times \frac{1000}{3600^2} = 3.25 \times 10^{-4}\ ms^{-2}$ ⇐

b) Her speed the instant she passes P

① V_Q = final velocity in trip 1 from P → Q

$V_Q = 2.4 + \frac{1}{2}(4.21)$ ② $s = ut + \frac{1}{2}at^2$ or $v^2 = u^2 + 2as$

$V_Q = 4.505\ Kmh^{-}$ $2.4 = u + \frac{1}{2}(4.21)$

Key for part ②, Final velocity from P to Q is = the initial velocity from Q to R

$u = 0.295\ Kmh^{-}$

$0.295 \times \frac{1000}{3600} = 0.082\ ms^{-1}$ ⇑

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Oct 2021 M1

A car is moving at a constant speed of 25 ms^{-1} along a straight horizontal road. At time $t=0$ the car is at point A, the driver sees a road sign 48m ahead. let t seconds be the time that elapses after the car passes A.

In the first model the car is assumed to decelerate uniformly at $6ms^{-2}$ from A to the road sign.

a) use first model to finds cars speed at the sign

$u = 25$ $\quad v^2 = u^2 + 2as$

$s = 48$ $\quad v^2 = (25)^2 + (-6)(48)2$

$a = -6$ $\quad v^2 = 625 + 576$

$v^2 = 49$

$v = 7\ ms^{-1}$ → speed at the sign.

In the second model, the car is assumed to decelerate uniformly at $6ms^{-2}$, from A until it reaches a speed of $B\ ms^{-1}$. The car then maintains this speed until reaching the road sign.

b) use second model to find the value of t at which the car reaches the sign.

① $-6 = \frac{13-25}{t}$

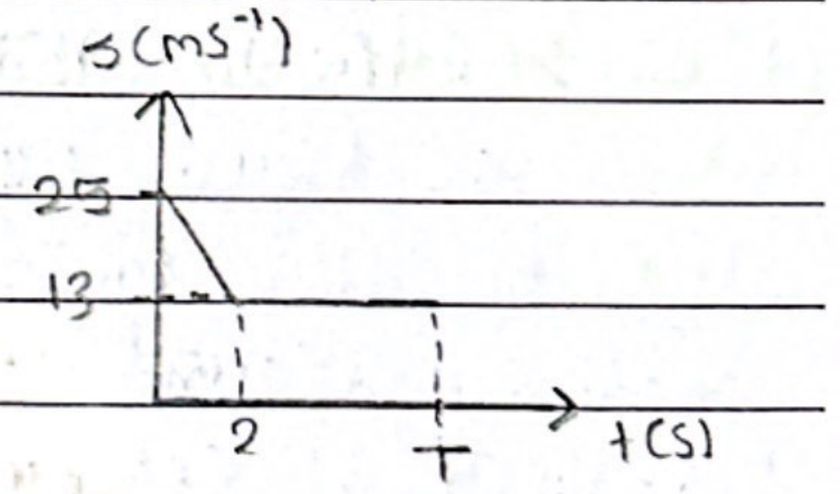
$t = 2$

② $48 = 13T + \frac{1}{2}(2)(12)$

$48 = 13T + 12$

$36 = 13T$

$\boxed{T = 2.77\ s}$



In a third model, the car is assumed to move with constant speed 25 ms^{-1} from A until $t = 0.2$, the car then decelerates uniformly at $6ms^{-2}$, until reaches sp of $13\ ms^{-1}$. The car then maintains this sp until reaching the sign.

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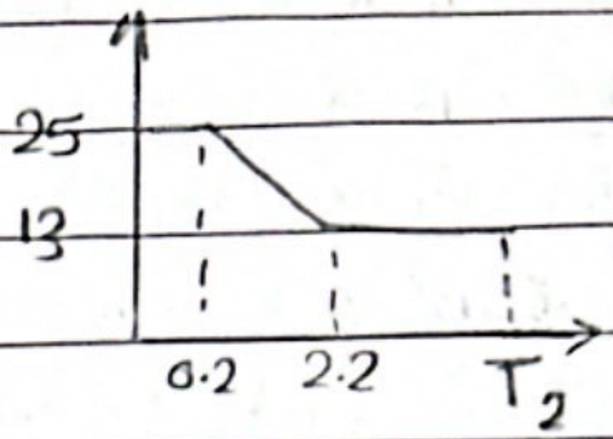
A) Use the third model to find the value of t_2 at which the car reaches the road sign.

$48 = 13T_2 + \frac{1}{2}(0.2+2.2)12$

$48 = 13T_2 + 14.4$

$33.6 = 13T_2$

$\boxed{T_2 = 2.585} \Leftarrow$



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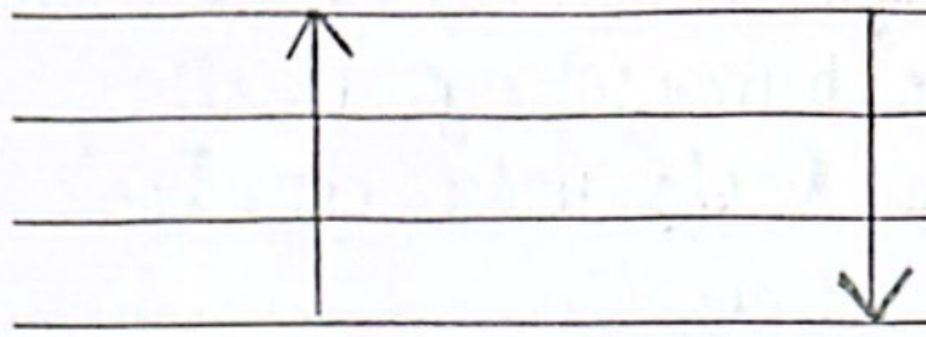
Vertical projectiles :-

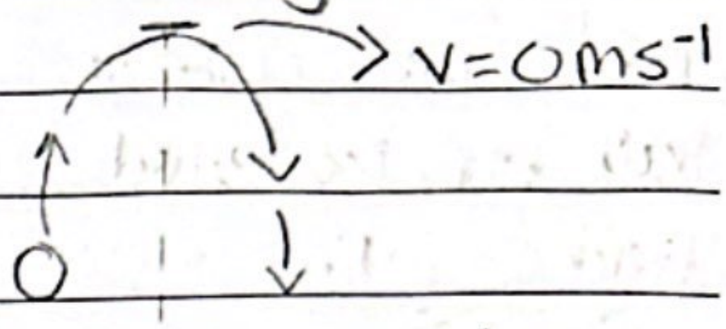
upwards

$a = -g = -9.8\,ms^{-2}$

downwards

$a = g = 9.8\,ms^{-2}$



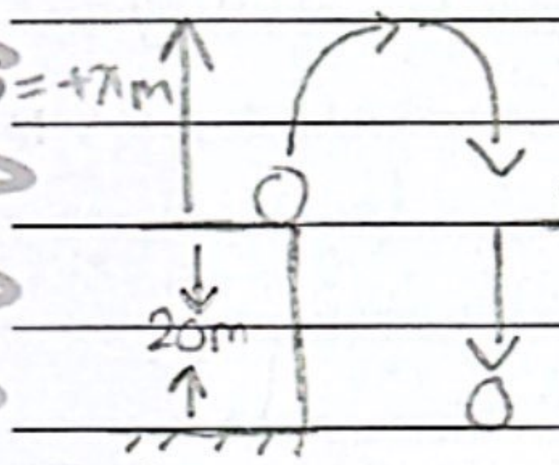


* under ideal conditions) the incline half is identical to decline one.

$t_{incline} = t_{decline}$

$s_{incline} = s_{decline\ to\ initial\ position}$

* In case of finding v immediately before impact with ground. Total displacement = - Elevated height m



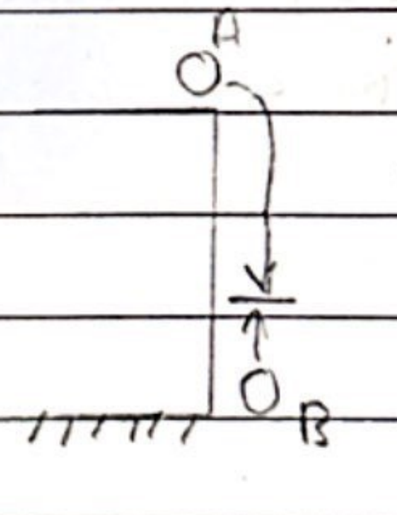
$s_{net} = 7 - 7 - 20$

$= -20m \Leftarrow$

* Same principle above ^ applies when getting the Total time

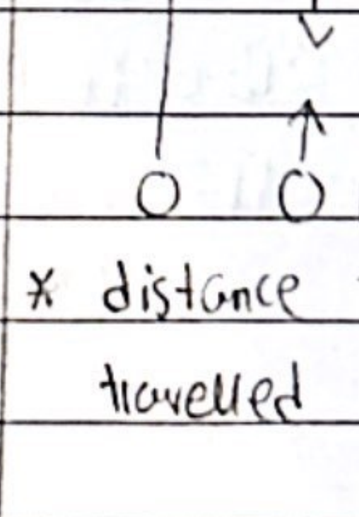
* Total Distance = 2(max height) + Elevated height

Colliding projectiles :-



$s_A + s_B$ = Building height

s_A in terms of t + s_B in terms of t = Height of building.



* distance fallen by A + distance travelled by B = max height

s_A in terms of t + s_B in terms of t = Max Height

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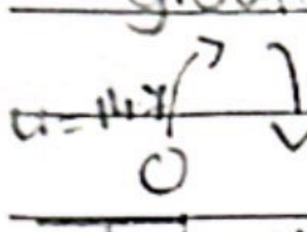
Example ⑦

A small ball is thrown vertically upwards with speed 14.7 ms^{-1} from a point that is 19.6 above horizontal ground. The ball is modelled as a particle moving freely under gravity.

Find :

a) Total time from when the ball is thrown till hitting ground.



$u = 14.7$ $\quad s = ut + \frac{1}{2}at^2$

$s = -19.6$ $\quad -19.6 = 14.7t + \frac{1}{2}(-9.8)t^2$

$a = -9.8$ $\quad -19.6 = 14.7t - 4.9t^2$

$4.9t^2 - 14.7t - 19.6 = 0$

$\Rightarrow t = 4$, $t = -1$ ✗ reject

b) The speed of the ball immediately before hitting the ground.

$u = 14.7$ $\quad v^2 = u^2 + 2as$

$s = -19.6$ $\quad v^2 = 216.09 + 2(-9.8)(-19.6)$

$a = -9.8$ $\quad v^2 = 600.25$

$v = 24.5\ ms^{-1} \Leftarrow$

c) The Total distance travelled by the ball from when its thrown till it hits the ground.

① Max height ?

$v = 0$ $\quad v^2 = u^2 + 2as$

$u = 14.7$ $\quad 0 = 216.09 - 19.6s$

$a = -9.8$ $\quad s = 11.025$

② $2(11.025) + 19.6$

$= 41.65\ m \Leftarrow$

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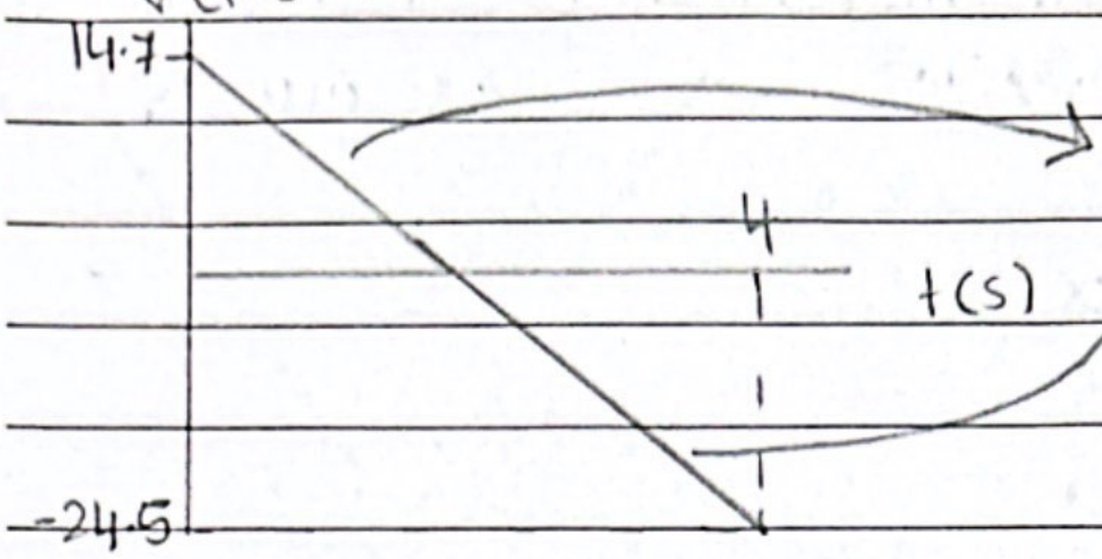
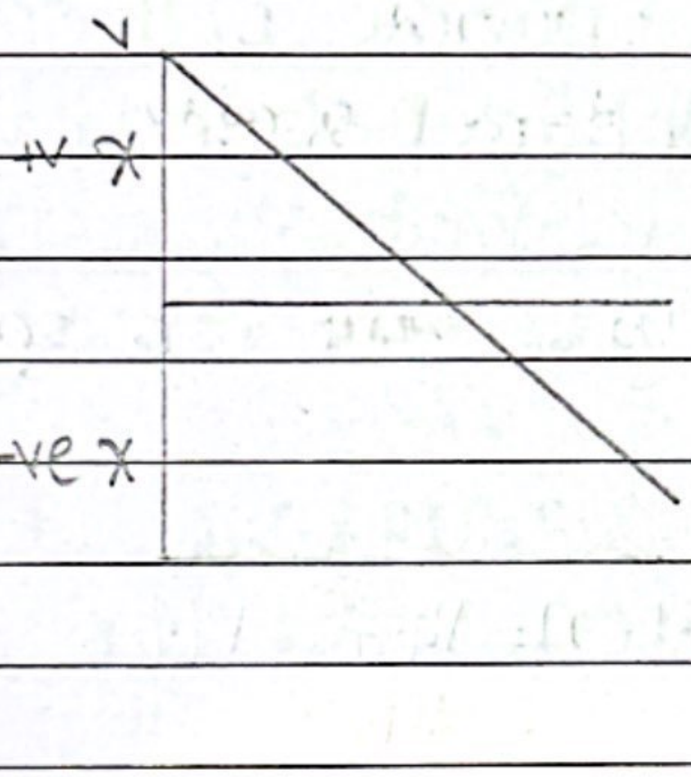
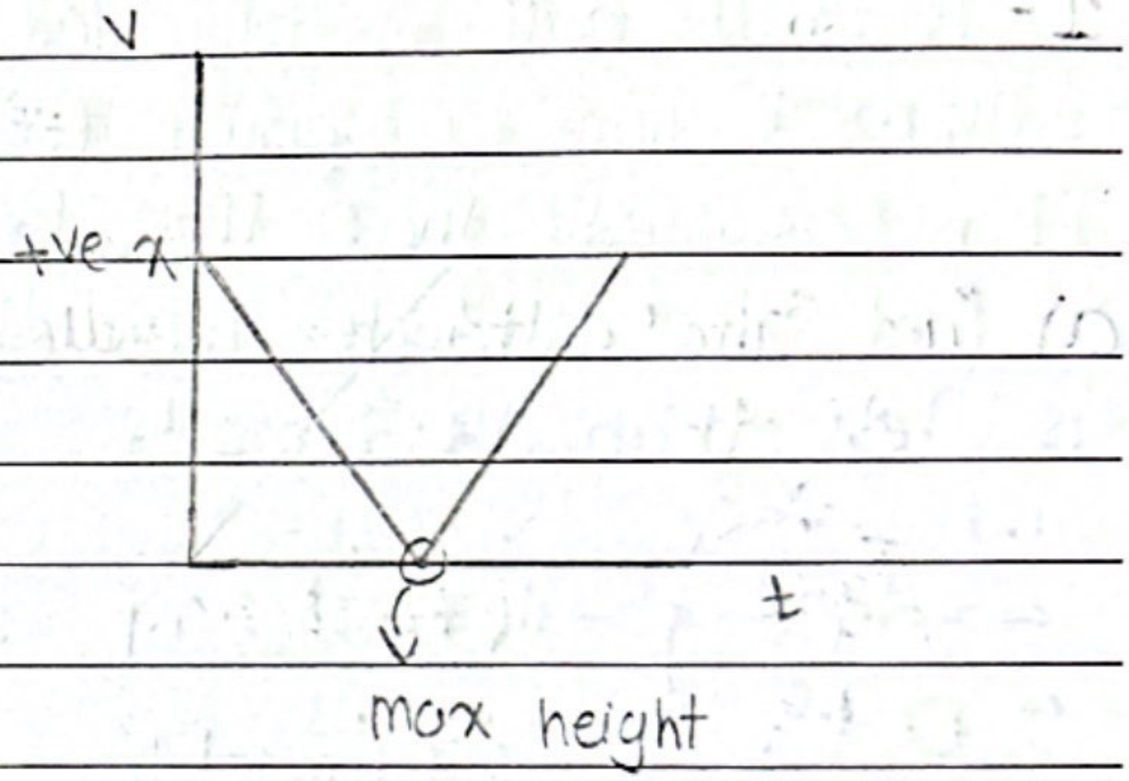
d) Sketch a velocity-time graph for the motion of the ball from when its thrown till it hits the ground-



notice how both lines have same gradient, since acceleration is always = g = ±9.8 ms⁻².

velocity-time vs speed-time

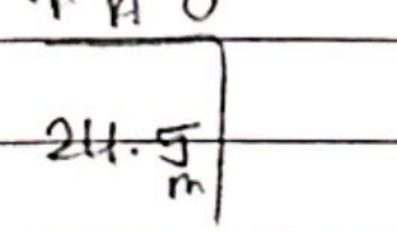




Example ⑧

At time $t=0$ a small ball is projected vertically upwards from a point A which is 24.5 m above ground. The ball first comes to instantaneous rest at B where AB = 19.6 m and first hits ground at $t=T$.

a) find T ①



② $u=19.6$, $s=-24.5$, $a=-9.8$

$v=0$	$v^2=u^2+2as$	$-24.5=19.6t-4.9t^2$
$s=19.6$	$0=u^2-384.16$	$4.9t^2-19.6t-24.5=0$
$a=-9.8$	$u=19.6\ ms^{-1}$	$t=5$, $t=-1$

$T=5s$ ⇐

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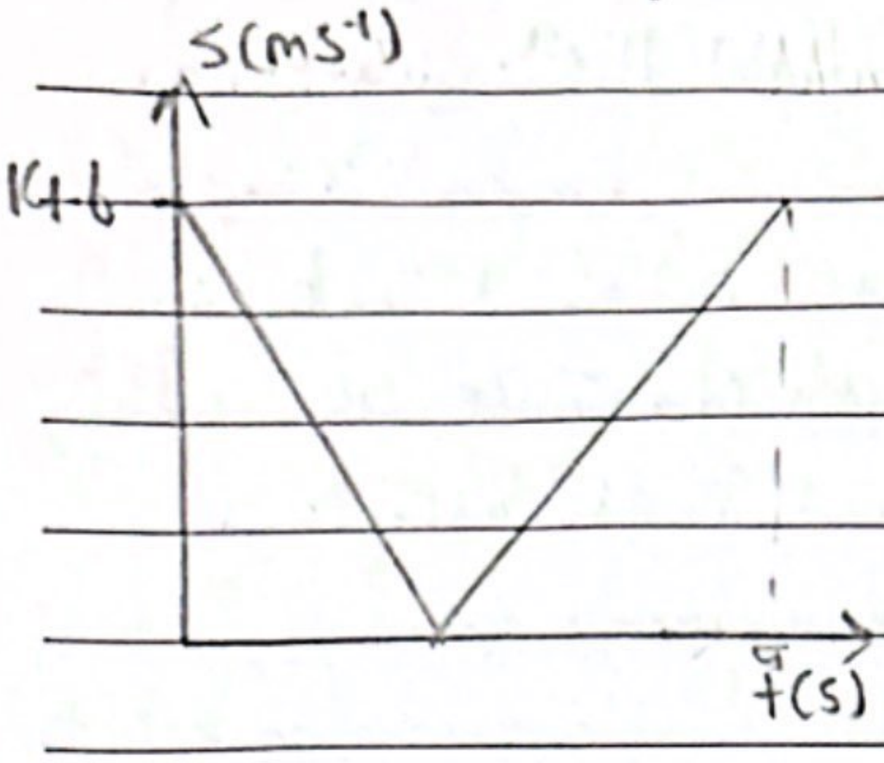
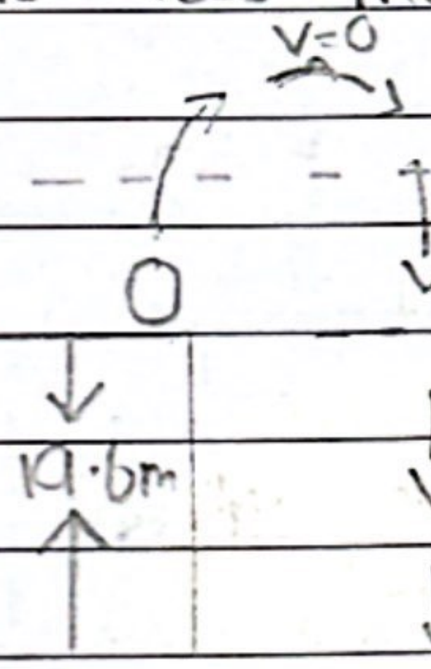
b) Sketch a speed -time graph from $t=0$ to $t=T$.



2 new/important ideas 8-

1- A small ball is projected vertically upwards with speed 29.4 ms⁻¹ from a point A which is 19.6 m above ground. It is assumed that the ball doesn't rebound.

a) find the distance travelled by the ball while its speed is less than 14.7 ms⁻¹.



$u = 14.7$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = 14.7^2 - 19.6s$

$a = -g$ $\quad s = \frac{14.7^2}{19.6}$

$s = 11.025$

$2 \times s = 20.05\text{ m} \Leftarrow$

b) find the time for which the ball is moving with a speed more than 29.4 ms⁻¹

$u = 29.4$ $\quad s = ut + \frac{1}{2}at^2$

$s = 19.6$ $\quad 19.6 = 29.4t + 4.9t^2$

$a = g$ $\quad 4.9t^2 + 29.4t - 19.6 = 0$

$\Rightarrow t = 0.61$, $t = -6.6$ ✗ reject

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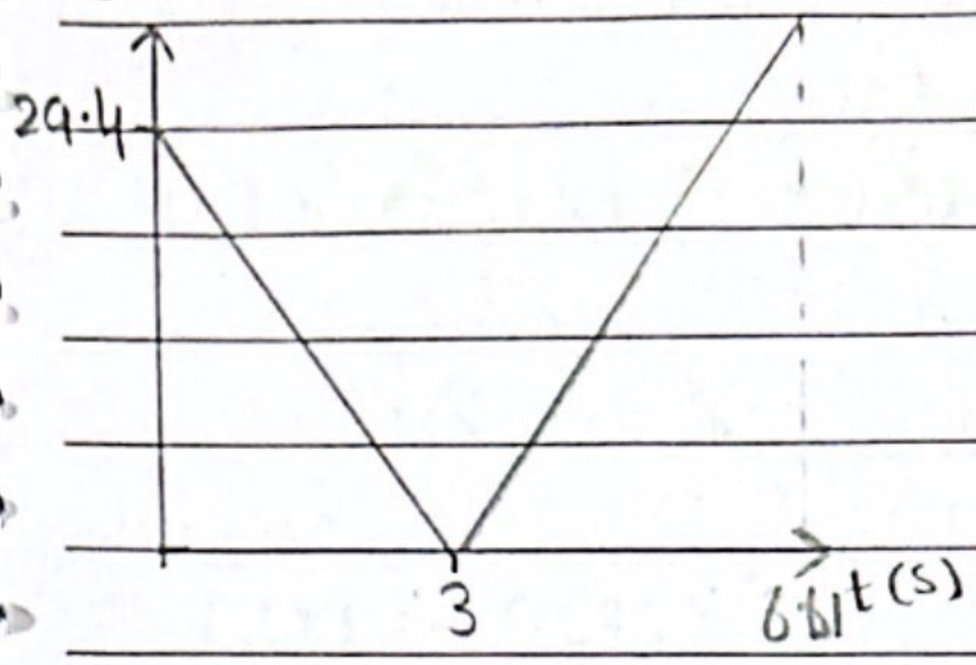
C) sketch a speed-time graph for the motion of the ball from point of projection to impact with ground.



2- A motorbike is moving with constant acceleration along a straight horizontal road. The motorbike passes a point P and 10s later passes a point Q. The speed of motorbike at Q is 28 ms^{-1}, Given PQ = 220m

a) find the acceleration of the motorbike

$s = 220$ $\quad s = vt - \frac{1}{2}at^2$

$t = 16$ $\quad 220 = 280 - 50a$

$v = 28$ $\quad 50a = 60$

$a = 1.2\ ms^{-2}$ ⇐

b) find the distance travelled by the motorbike during the fifth second after passing P.

5^{th} second = from 4^{th} second to 5^{th} second

Displacement for $t = 5$ − displacement for $t = 4$ $= s_{5^{th}}$

$u = 16$, $t = 5/4$, $a = 1.2$ $\quad s = vt + \frac{1}{2}at^2$

$s_5 = 16(5) + \frac{1}{2}(1.2)5^2$

$s_5 = 95$ $\quad 95 - 73.6 = 21.4\ m$ ⇐

$s_4 = 16(4) + \frac{1}{2}(1.2)4^2$

$= 73.6$

N.B

Somthing in n seconds = from $t = 0$ to $t = n$

somthing at n^{th} second = from $t = (n-1)$ to $t = n$

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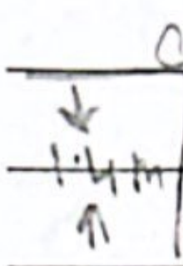
Textbook questions 8-

1- A book falls off the top of the shelf. The shelf is 1.4 m above wooden floor. Find :-

a) the time the book takes to reach the floor



$u=0$, $s=1.4$, $a=g$

$s = ut + \frac{1}{2}at^2$

$1.4 = 4.9t^2$

$t^2 = \frac{2}{7}$ $\quad t = 0.535$ ⇐

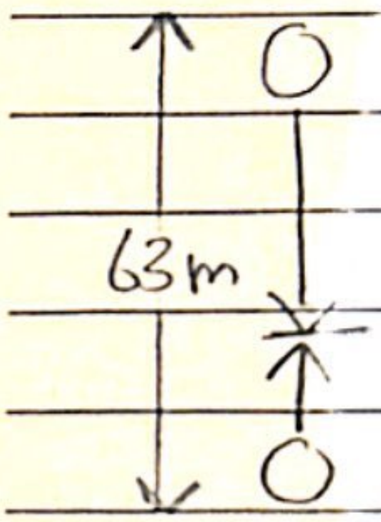
b) the speed at which the book strikes the floor.

$v^2 = u^2 + 2as$

$v^2 = 0 + 27.44$

$v = 5.2$ ⇐

2- A Ball falls vertically from rest at the top of a tower 63 m high. At the same time as A begins to fall, another ball B is projected vertically upward from the bottom of the tower with speed 21 ms^{-1}. The balls collide. Find the distance of the point where the balls collide from the bottom of the tower.



	for A	for B
	$u = 0$	$u = 21$
	$a = g$	$a = -g$

Plong :-

① get each s_A and s_B in terms of time

② equate the sums with 63m ±SBC

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$s = ut + \frac{1}{2}at^2$

$s_A = \frac{1}{2}(9.8)t^2$ $\quad s_B = 21t + \frac{1}{2}(9.8)t^2$

$s_A = 4.9t^2$ $\quad s_B = 21t - 4.9t^2$

$s_A + s_B = 63$

$4.9t^2 + 21t - 4.9t^2 = 63$

$21t = 63$

$t = 3$ seconds ←

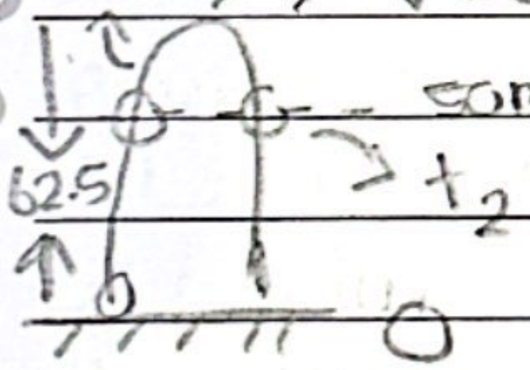
distance from bottom $= s_B$

$= 21(3) - 4.9(3)^2$

$= 18.9$ m ⇐

3- A particle is projected vertically upwards from a point O with speed u ms^{-1}. The greatest height reached by the particle is 62.5 m above O. Find!

a) value of u



$v = 0$ $\quad s = 62.5$ $\quad a = -g$

$v^2 = u^2 + 2as$

$0 = u^2 - 1225$

$u = 35$ ⇐

b) the total time for which the ball is 50m above O.

$u = 35$ $\quad s = 50$ $\quad a = -g$

$s = ut + \frac{1}{2}at^2$

$50 = 35t - 4.9t^2$

$4.9t^2 - 35t + 50 = 0$

$t_1 = 1.97$, $t_2 = 5.17$

$t_2 - t_1 = 3.2$ s ⇐

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All variables Qn 8- Jan 2020

A particle P, is projected vertically upwards with speed U from a fixed point O. At the instant when P reaches its greatest height H above O, a second particle Q is projected with speed $\frac{1}{2}U$ vertically upwads from O

a) Find H in terms of U and g

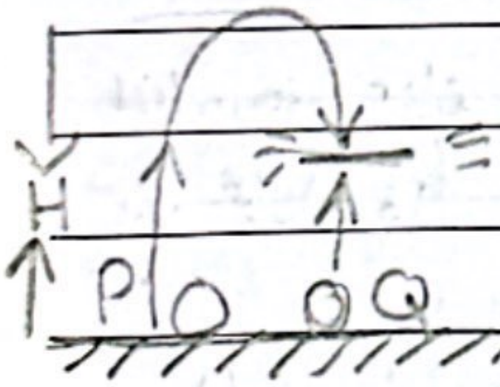
$s = H$ $\quad v^2 = u^2 + 2as$

$v = 0$ $\quad 0 = U^2 - 2gH$

$u = U$ $\quad 2gH = U^2$

$a = -g$ $\quad H = \frac{U^2}{2g}$

b) Find in terms of U ang g the time between the instant when Q is projected and the instant when the two particles collide



$H = s_{P \text{ from max}} + s_Q$

for P	for Q
$u = 0$	$u = \frac{1}{2}U$
$a = g$	$a = -g$

$s = ut + \frac{1}{2}at^2$

$s_P = \frac{1}{2}gt^2$ $\quad s_Q = \frac{1}{2}ut - \frac{1}{2}gt^2$

$H = \frac{1}{2}gt^2 + \frac{1}{2}ut - \frac{1}{2}gt^2$

$H = \frac{1}{2}ut \rightarrow$ substitute $\frac{U^2}{2g} = H$

$\frac{1}{2}ut = \frac{u^2}{2g}$ $\quad (\times 2)$

$ut = \frac{u^2}{g}$

$t = \frac{U}{g} \Leftarrow$

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c) Find where the two particles collide
= displacement from ground
= s_Q

$s_Q = \frac{1}{2}ut - \frac{1}{2}gt^2 \rightarrow t = \frac{u}{g}$

$s_Q = \frac{1}{2}u\left(\frac{u}{g}\right) - \frac{1}{2}g\left(\frac{u}{g}\right)^2$

$s_Q = \frac{1}{2}\frac{u^2}{g} - \frac{1}{2}g\left(\frac{u^2}{g^2}\right)$

$s_Q = \frac{u^2}{2g} - \frac{u^2}{2g}$

s_Q = zero $\Leftarrow$ P collides with Q before it even has a displacement.

A random 17 marker :-

Two cars, A and B move on parallel straight tracks. initially A and B are at rest with A at P and B at Q. At time $t = 0$. A starts to move with constant acceleration $a\ ms^{-2}$ for 3.5 s reaching a speed of $14\ ms^{-1}$. Car A then moves with constant speed $14\ ms^{-1}$.

a) find the value of a

$a = \frac{v-u}{t} \quad a = \frac{14-0}{3.5} \quad a = 4 \Leftarrow$

Car B also starts to move at $t = 0$ in same direction as A. Car B moves with constant acceleration of $3\ ms^{-2}$. At $t = T$ B overtakes A. At the instant A was moving with constant speed.

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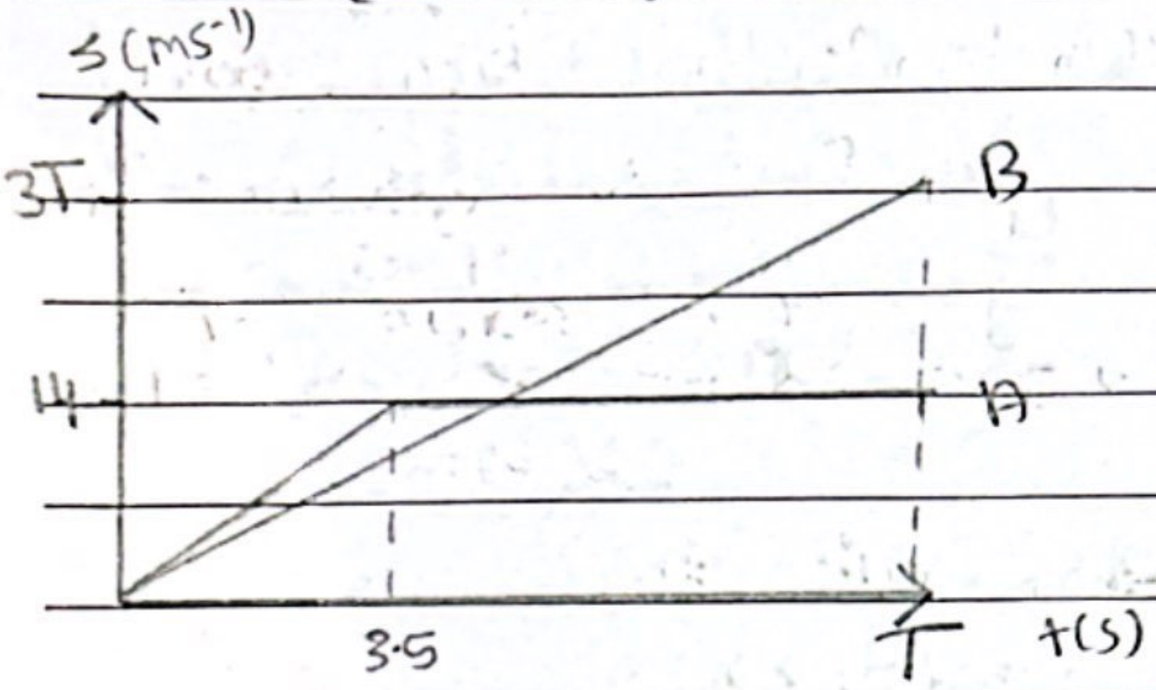
b) sketch on the same axis a speed-time graph for A and B. until $t = T$



c) Find the value of T for car B

$u = 0$ $v = u + at$

$t = T$ $v = 3T$ ←

$a = 3$

$s_A = s_B$

$s_A = \frac{1}{2}(T+T-3.5)14$

$= 14T - 24.5$ → ①

$s_B = \frac{1}{2}(T)(3T)$

$= \frac{3}{2}T^2$ → ②

$14T - 24.5 = \frac{3}{2}T^2$

$\frac{3}{2}T^2 - 14T + 24.5 = 0$

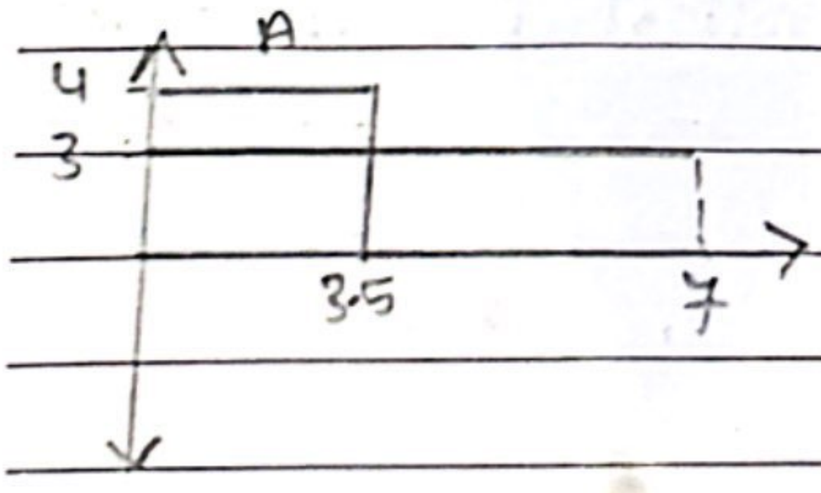
$T = \frac{7}{3}$, $T = 7$ ✓ ⇐

x reject as $\frac{7}{3} < 3.5$

d) find distance of B from Q when B overtakes A

$\frac{3}{2}(7)^2 = 73.5$ m ⇐

e) Sketch an acceleration-time graph



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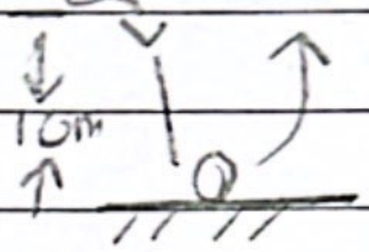
Textbook questions 8-

1- A ball is released from rest from a point which is 10m above floor. Each time The ball strikes the floor it rebounds with three-quarters of the speed with which it strikes the floor. Find the greatest height above floor reached by the ball :-

a) the first time it rebounds off the floor.



① $u = 0$ $\quad v^2 = u^2 + 2as$

$s = 10$ $\quad v^2 = 196$

$a = 9.8$ $\quad v = 14\ ms^{-1}$

②

speed of rebound $= \frac{3}{4} \times 14$

$= 10.5\ ms^{-1}$

$u = 10.5$ $\quad v^2 = u^2 + 2as$

$a = -9.8$ $\quad 0 = 110.25 - 19.6s \rightarrow s = 5.625m \Leftarrow$

$v = 0 \rightarrow$ At max

b) the second time it rebounds off the floor

speed of second impact is $10.5\ ms^{-1} \rightarrow$ by symmetery

speed of rebound $= 3/4 \times 10.5$

$= 7.875\ ms^{-1}$

$u = 7.875$

$a = -9.8$ $\quad 0 = 7.875^2 - 19.6s$

$v = 0$ $\quad s = 3.2\ m \Leftarrow$

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2- A particle P is moving along the x-axis with constant deceleration $2.5\ ms^{-2}$. At time $t=0$ P passes through origin O with velocity $20\ ms^{-1}$ in the direction of x increasing. At $T=12$ P is at A, find:

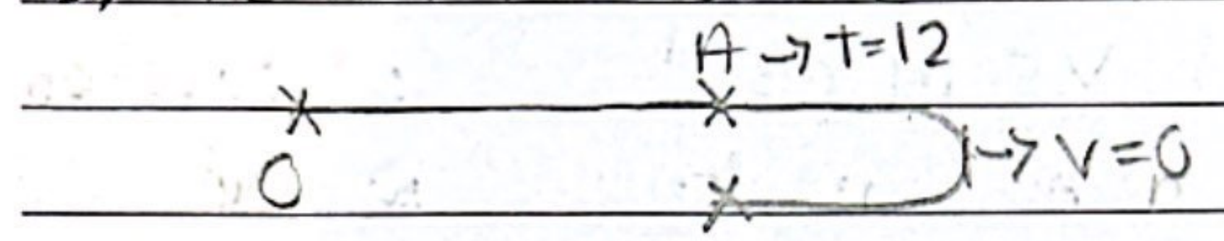
a) The distance OA

$u=20$ $\quad s = ut + \frac{1}{2}at^2$

$a=-2.5$ $\quad s = (20\times12) + \frac{1}{2}(-2.5)(12)^2$

$t=12$ $\quad s = 60\ m \Leftarrow$

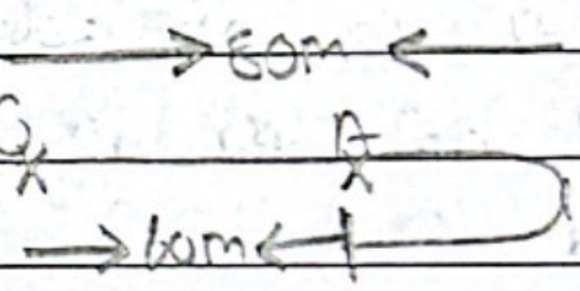
b) The total distance P travels in 12s



$u=20$ $\quad v^2 = u^2 + 2as$ $\quad$ 80 m from O to max point

$a=-2.5$ $\quad 0 = 400 - 5s$

$v=0$ $\quad s = 80\ m$



$80 + (80-60)$

$= 100\ m \Leftarrow$

3- A pebble is projected vertically upward with speed $21\ ms^{-1}$ from a point 32 m above ground. Find

a) the speed which the pebble touches ground.

$s=-32$ $\quad v^2 = u^2 + 2as$

$a=-9.8$ $\quad v^2 = 21^2 + (-9.8)(-32)(2)$

$u=21$ $\quad v^2 = 1068.2$

$v = 32.7 \Leftarrow$

b) The total time for which the pebble is more than 40m above ground.

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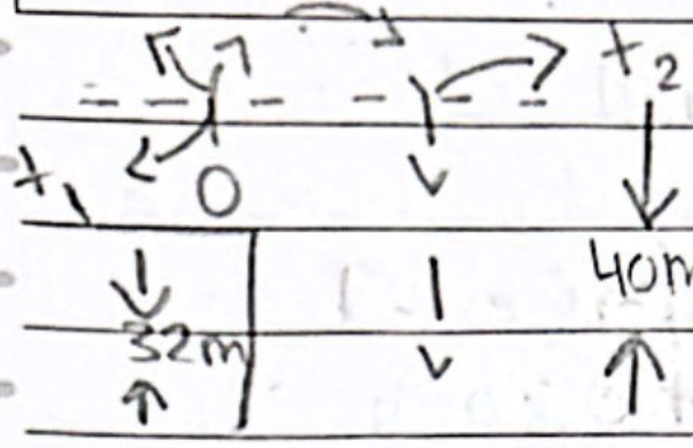
8 m above the 32 m



$s = 8$ $\quad s = ut + \frac{1}{2}at^2$

$a = -9.8$ $\quad 8 = 21t - 4.9t^2$

$u = 21$ $\quad t = 3.86, t = 0.42$
$\quad\quad\quad

(- Time when B overtakes A.

s_A at time $= T$ $=$ s_B at time $= T$

$u = 20$, $v = 32$, $a = 4$

$s_A = 30T$

$s_B = 20T + 2T^2$

$30T = 20T + 2T^2$

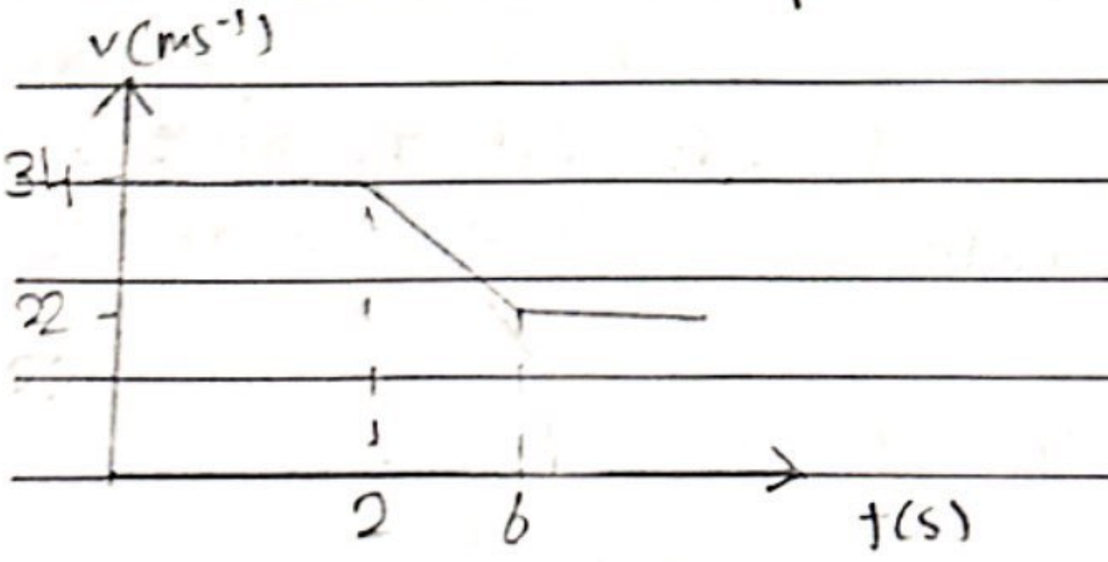
$2T^2 - 10T = 0$

$2T(T - 5) = 0$

$T = 5$ ⇐

5- A car is being driven on a straight stretch of motorway at constant velocity of $34\ ms^{-1}$ when it passes a velocity restriction sign S warning of road works ahead and requiring speed reduction to $22\ ms^{-1}$. The driver continues at her velocity for 2s after passing S. She then reduces her velocity to $22\ ms^{-1}$ with constant deceleration of $3\ ms^{-1}$, and continues at the lower velocity

a) Draw a velocity-time graph to show the motion of the car after it passes S.



b) find the shortest distance before the roadworks that S should be placed to ensure that a car driven in this way has had its velocity reduced to $22\ ms^{-1}$ by the time it reaches the road works.

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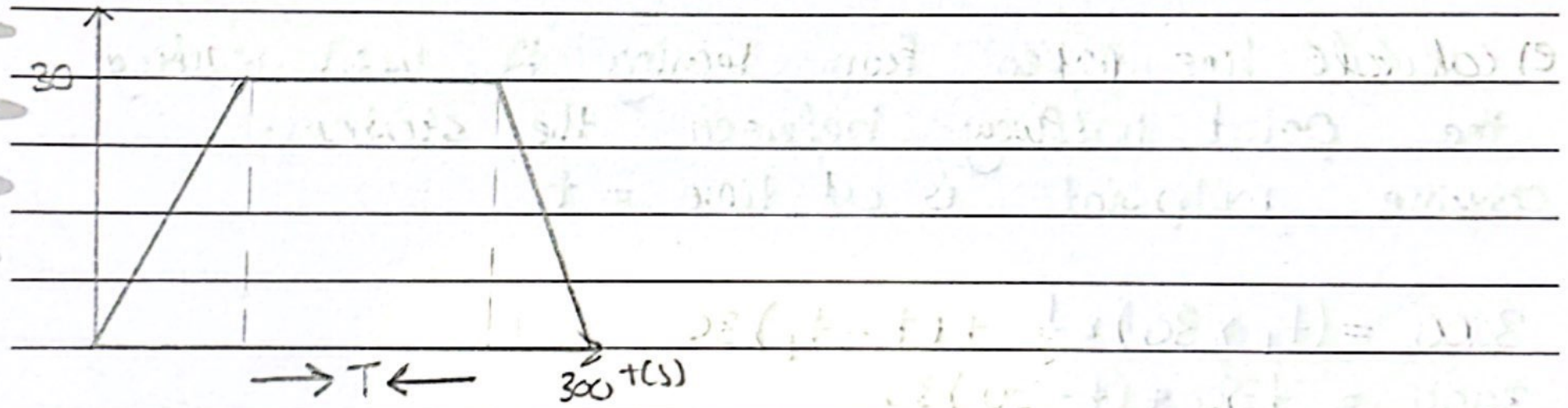
Plan is to find the distance that took the car above b seconds to reach 22 ms-1.

Area under graph = $(2 \times 34) + \frac{1}{2}(22+34)4$

$= 180m \Leftarrow$

6- A train strarts from rest at station A and accelerates uniformly at $3\lambda\ ms^{-2}$ until it reaches a velocity of 30ms-1. For the next T seconds the train maintains the constant velocity. The train then decelerates uniformly at $\lambda\ ms^{-2}$ until it comes to rest at station B. The distance between the stations is 6km and time taken from A to B is 5 mins.

a) Sketch a velocity-time graph



b) Show that $\frac{40}{\lambda} + T = 300$

300 = Total time

$\text{time}_1 = \frac{30-0}{3\lambda}$ $\quad \text{time}_2 = \frac{0-30}{-\lambda}$

$t_1 = \frac{10}{\lambda}$ $\quad t_2 = \frac{30}{\lambda}$

$300 = \frac{10}{\lambda} + \frac{30}{\lambda} + T$

$300 = \frac{40}{\lambda} + T$, as required

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c) Find the value of x and T

①

$6000m = \frac{1}{2}(300 + T)30$

$6000 = 4500 + 15T$

$1500 = 15T$

$T = 100 \Leftarrow$

②

$300 = T + \frac{40}{x}$

$300 = 100 + 40x^{-}$

$200 = 40x^{-}$

$x = \frac{40}{200}$

$x = 0.2 \Leftarrow$

d) calculate the distance travelled at constant velocity

$T \times 30$

$= 30(100)$

$= 3000\ m$

$= 3\ Km \Leftarrow$

e) Calculate time taken from leaving A until reaching the point halfway between the stations.

assume midpoint is at time $= t$

$3000 = (t_1 \times 30) \times \frac{1}{2} + (t - t_1)30$

$3000 = 750 + (t - 50)30$

$2250 = 30t - 1500$

$3750 = 30t$

$t = 125\ s \Leftarrow$

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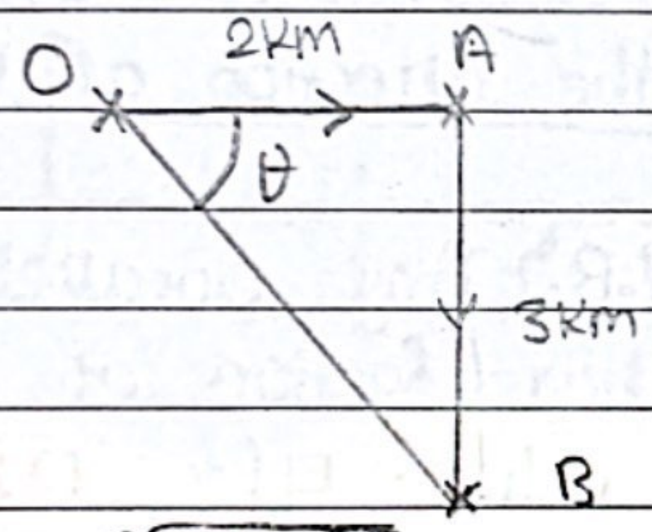
Chapter 2 Vectors

Determining magnitude and direction :-

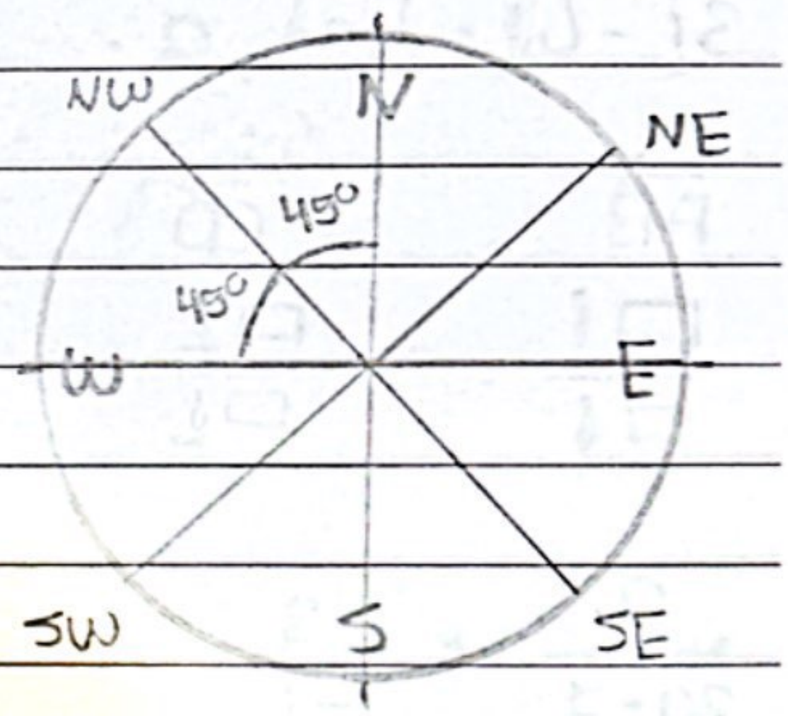
Magnitude → apply pythagoras' theorem
Direction → Bearing from origin / required point

Worked example ①

A girl walks 2Km due east from a fixed point O to A, and then 3Km due south from A to B, find the total distance walked and describe displacement B from O.



Know your compass:



$|\overrightarrow{OB}| = \sqrt{2^2+3^2}$

$|\overrightarrow{OB}| = 3.61$ Km ←

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}(3/2)$

$\theta = 56.3°$

Bearing from O = 56.3 + 90

= 146.3 ←

Magnitude using $\underline{i}$ and $\underline{j}$ notation :-

$\overrightarrow{AB} = a\underline{i} - b\underline{j}$

$|\overrightarrow{AB}| = \sqrt{(a)^2+(-b)^2}$

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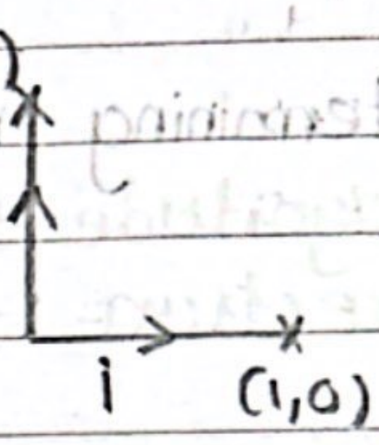
$\underline{i}$ and $\underline{j}$ unit vector notation :-



$\underline{i}$ → Horizontal movement of 1 unit
$\underline{j}$ → Vertical movement of 1 unit

Vectors in parallel :-

Condition :- either vector has to be a multiple of the other. or a

$\overrightarrow{AB} = K\overrightarrow{CD}$, where K is an integer × fraction

→ = Parallel

Worked example ②

$\overrightarrow{AB} = 2\underline{i} + (2a-7)\underline{j}$ is moving in the direction of $\overrightarrow{CD} = 3\underline{i} - 6\underline{j}$. Find a.

(N.B.) Any parallel vectors have fractions of equal ration. $\frac{\square\underline{i}}{\square\underline{j}} = \frac{\square\underline{i}}{\square\underline{j}}$ ($\vec{A}$ = $\vec{B}$)

$\frac{\overrightarrow{AB}}{\square\underline{j}} = \frac{\overrightarrow{CD}}{\square\underline{j}}$

$\frac{2}{2a-7} = \frac{3}{-6}$

$-12 = 6a - 21$

$6a = 9$

$a = 3/2$ ⇐

Worked example ③

Given that $p = 4\underline{i} + 3\underline{j}$ and $q = 2\underline{i} - 6\underline{j}$, find λ if $p + \lambda q$ is parallel to the vector

a) $\underline{j}$

$p + \lambda q = 4\underline{i} + 3\underline{j} + \lambda(2\underline{i} - 6\underline{j})$

$= 4\underline{i} + 3\underline{j} + 2\lambda\underline{i} - 6\lambda\underline{j}$

$= (4 + 2\lambda)\underline{i} + (3 - 6\lambda)\underline{j}$

new vector

$\frac{4+2\lambda}{3-6\lambda} = \frac{0}{1}$ ← no $\underline{i}$ component

$4 + 2\lambda = 0$

$2\lambda = -4$

$\lambda = -2$ ⇐

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b) $\underline{i} + \underline{j}$

$$\frac{4+2\lambda}{3-6\lambda} = \frac{1}{1}$$

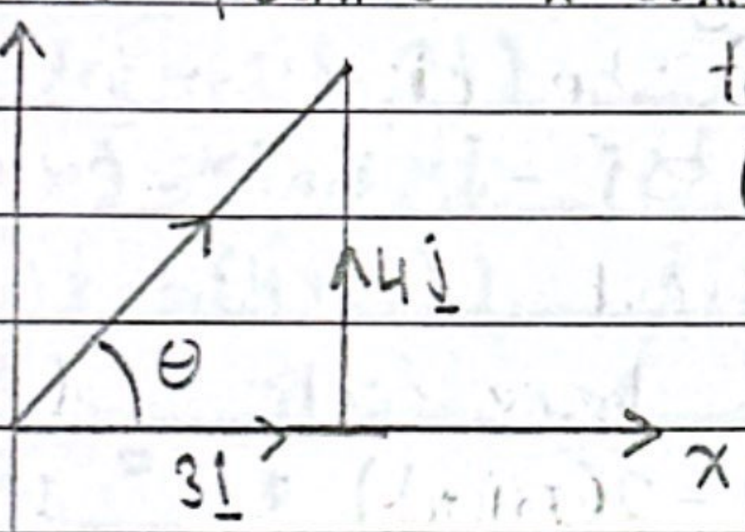
$$4+2\lambda = 3-6\lambda$$

$$8\lambda = -1$$

$$\lambda = -1/8 \Leftarrow$$

Angles a vector makes with x / y axis :-

Vector $a = 3\underline{i} + 4\underline{j}$, find the angle this vector makes with the positive x-axis.

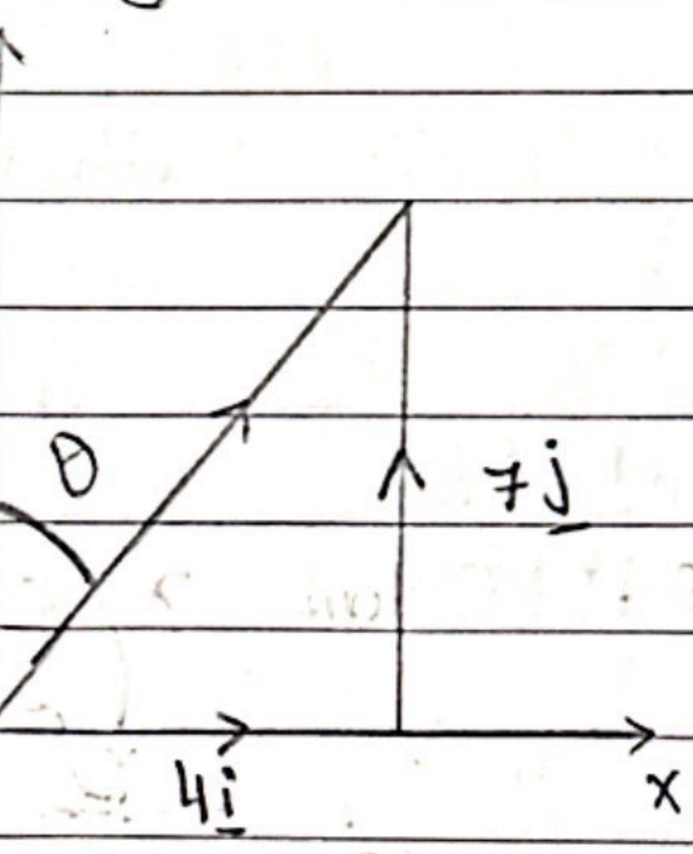


$$\tan\theta = \frac{4}{3}$$

$$\theta = \tan^{-1}(4/3)$$

$\theta = 53.1^{\circ}$ above x-axis

Vector $b = 4\underline{i} + 7\underline{j}$, find the angle it make with the positive y axis.



$$\tan\theta = 4/7$$

$$\theta = \tan^{-1}(4/7)$$

$\theta = 29.7^{\circ}$ to the right

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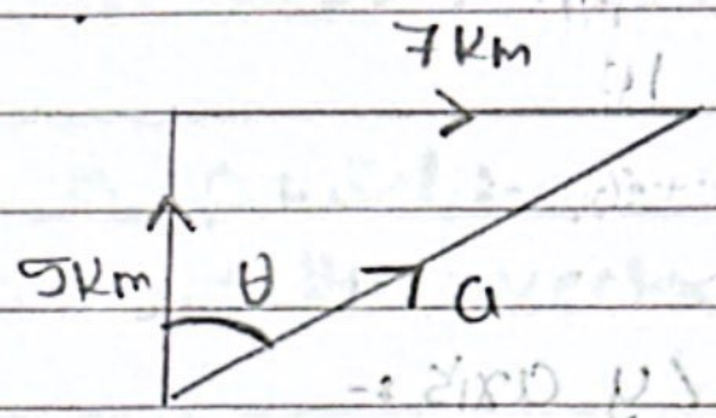
Text book Qns :-

1- A Bird flies 5 Km due north and then 7 Km due east. How far is the bird from original position and in what direction ?



$|a| = \sqrt{5^2 + 7^2}$

$|a| = 8.6$ Km

$\tan\theta = 7/5$

$\theta = \tan^{-1}(7/5)$

$\theta = 54.5°$ due north

Bearing $= 054°$

2- $\underline{a} = 2\underline{i} + 3\underline{j}$, $\underline{b} = 3\underline{i} - 4\underline{j}$, $\underline{c} = 5\underline{i} - \underline{j}$ Find exact magnitude of :

a) $\underline{a} + \underline{b}$

$2\underline{i} + 3\underline{j} + 3\underline{i} - 4\underline{j}$

$= 5\underline{i} - \underline{j}$

$|\underline{a} + \underline{b}| = \sqrt{5^2 + (-1)^2}$

$= \sqrt{26}$ ⇐

b) $3\underline{b} - 2\underline{c}$

$3(3\underline{i} - 4\underline{j}) - 2(5\underline{i} - \underline{j})$

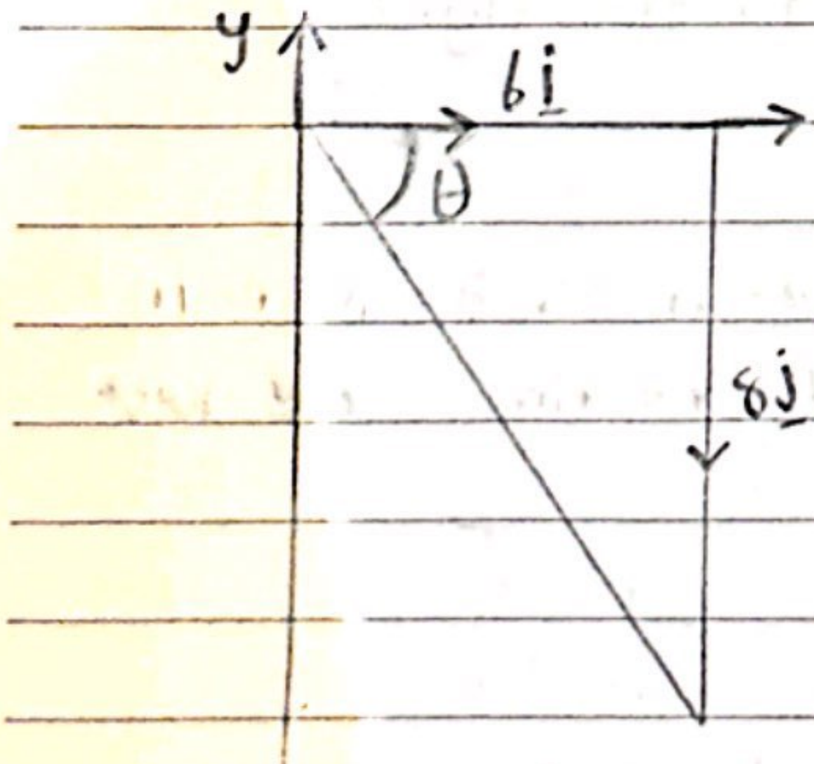
$9\underline{i} - 12\underline{j} - 10\underline{i} + 2\underline{j}$

$-\underline{i} - 10\underline{j}$

$|3\underline{b} - 2\underline{c}| = \sqrt{(-1)^2 + (-10)^2}$

$= \sqrt{101}$ ⇐

3- Find the angle that $6\underline{i} - 8\underline{j}$ make with +ve x-axis



$\tan\theta = 8/6$

$\theta = \tan^{-1}(8/6)$

$\theta = 53.1°$ Below x-axis

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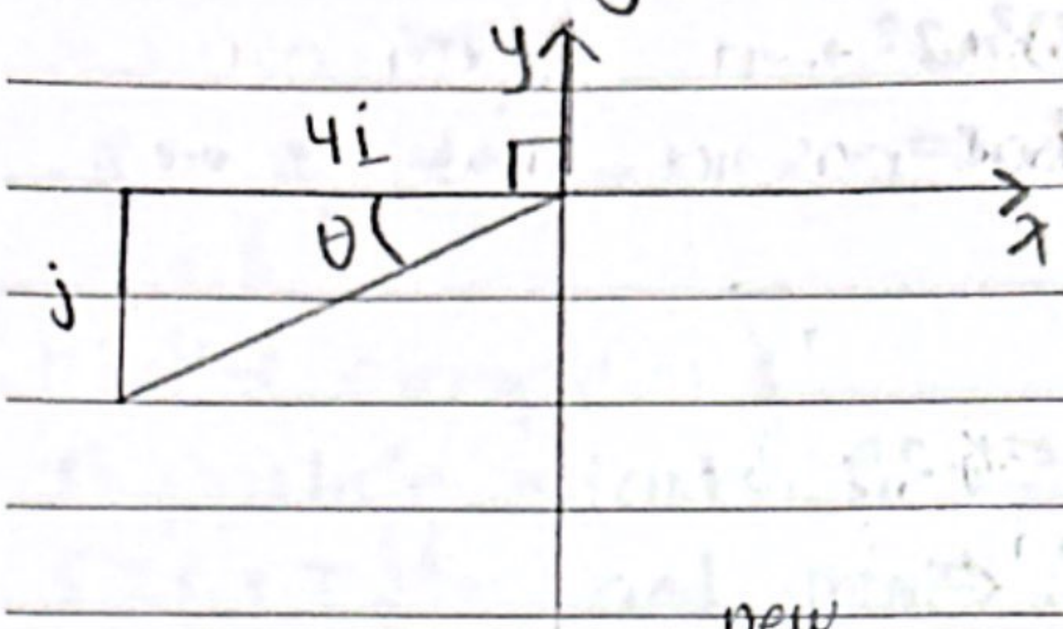
4- Find the angle that $-4\underline{i} - \underline{j}$ makes with +ve y axis



$\tan\theta = \frac{1}{4}$

$\theta = \tan^{-1}(1/4)$

$\theta = 14^\circ$

$14 + 90 = 104$

104° to the left

5- Given that $c = 3\underline{i} + 4\underline{j}$ and $d = \underline{i} - 2\underline{j}$ Find:

a) s if $c - sd$ is parallel to $2\underline{i} + \underline{j}$

$3\underline{i} + 4\underline{j} - s(\underline{i} - 2\underline{j})$ → new vector

$3\underline{i} + 4\underline{j} - s\underline{i} + 2s\underline{j}$

$(3 - s)\underline{i} + (4 + 2s)\underline{j}$

$\frac{3-s}{4+2s} = \frac{2}{1}$ → $3 - s = 8 + 4s$

$5s = -5$

$s = -1 \Leftarrow$

b) t if $\underline{d} - t\underline{c}$ is parallel to $-2\underline{i} + 3\underline{j}$

$\underline{i} - 2\underline{j} - t(3\underline{i} + 4\underline{j})$ → new vector

$= \underline{i} - 2\underline{j} - 3t\underline{i} - 4t\underline{j}$

$= (1 - 3t)\underline{i} - (2 + 4t)\underline{j}$

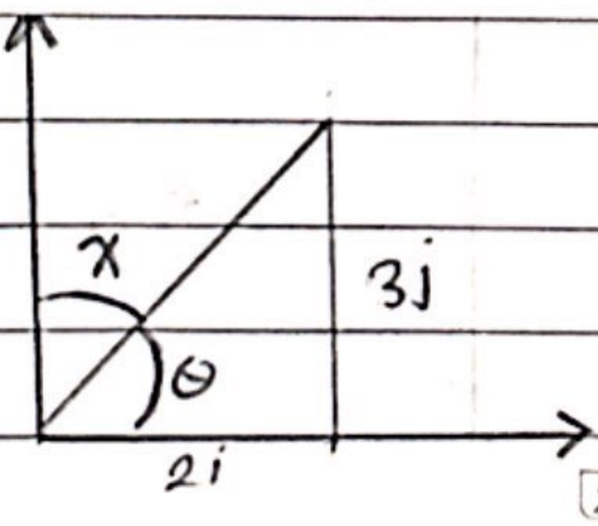
$\frac{1-3t}{-(2+4t)} = \frac{-2}{3}$ → $3 - 9t = 4 + 8t$

$-1 = 17t$

$t = -1/17 \Leftarrow$

5- In this Qn j is in north direction and i is in east direction. Find magnitude and bearing of these vectors-

a) $2\underline{i} + 3\underline{j}$



Magnitude $= \sqrt{2^2 + 3^2}$

$= 3.6 \Leftarrow$

Bearing $= 90 - \tan^{-1}(3/2)$

$= 034^\circ \Leftarrow$

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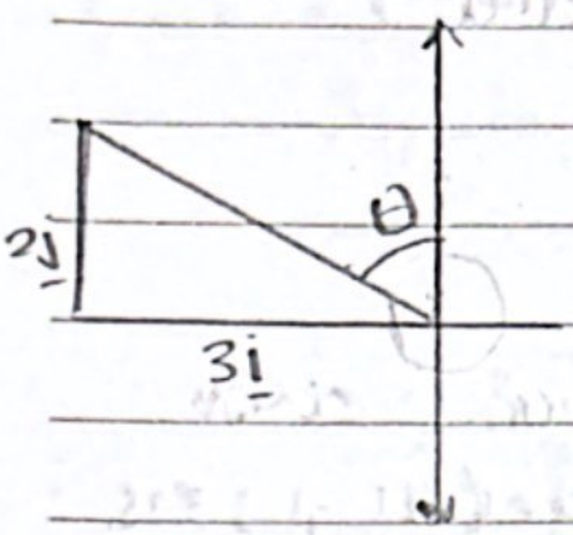
b) $-3\underline{i} + 2\underline{j}$



Magnitude $= \sqrt{(-3)^2 + 2^2}$

$= 3.6$ ⇐

$\theta = \tan^{-1}(3/2)$

$\theta = 56.3°$

Bearing $= 360 - 56.3$

$= 304°$ ⇐

Velocity and Displacement :-

|Velocity| = speed

|Displacement| = distance

Worked example ④ :

A particle is moving with constant velocity $v = (3\underline{i} + \underline{j})\,ms^{-1}$

Find :

a) The speed of the particle

$sp = |v|$

$|3i + j| = \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

$= 3.16\ ms^{-1}$

b) Distance moved in 4 s.

$D = S \times t$

$= 3.16 \times 4$

$= 12.6\ m$

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Final Position vector = Position vector (initial) + velocity x time (Displacement)

$$r = r_0 + vt$$

Worked example ⑤ :

A particle starts from the point with position vector $(3\underline{i} + 7\underline{j})$ m and moves with constant velocity $(2\underline{i} - \underline{j})$ ms^{-1}. Find the position vector of the particle 4s later.

$r_0 = 3\underline{i} + 7\underline{j}$

$v = 2\underline{i} - \underline{j}$

$t = 4$

$r = (3\underline{i} + 7\underline{j}) + (2\underline{i} - \underline{j}) \times 4$

$r = 3\underline{i} + 7\underline{j} + 8\underline{i} - 4\underline{j}$

$r = (11\underline{i} + 3\underline{j})$ m ⇐

Worked example ⑥ :

A particle is moving at a constant velocity is at position vector $(2\underline{i} - 4\underline{j})$ m at $t = 0$. Five seconds later it is at a point with position vector $(12\underline{i} + 16\underline{j})$ m. Find its velocity.

$r_0 = (2\underline{i} - 4\underline{j})$

$r = (12\underline{i} + 16\underline{j})$

$t = 5$

$(12\underline{i} + 16\underline{j}) = (2\underline{i} - 4\underline{j}) + 5v$

$$v = \frac{(12\underline{i} + 16\underline{j}) - (2\underline{i} - 4\underline{j})}{5}$$

$$v = \frac{10\underline{i} + 20\underline{j}}{5}$$

$v = (2\underline{i} + 4\underline{j})$ ms^{-1} ⇐

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Unit Vector :- is a vector that has a magnitude of 1

$\hat{a} = \frac{a}{|a|}$

e.g. $a = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ Find $\hat{a}$

$\hat{a} = \frac{1}{|a|} \times \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \frac{1}{\sqrt{3^2+4^2}} \times \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$= \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\hat{a} = \left(\frac{3}{5}, \frac{4}{5}\right)$ ⇐

How to change velocity into speed?

① Get the unit vector of direction vector

② velocity = unit vector of direction vector x speed

Worked example ⑦.

At time $t=0$, a particle has position vector $4\underline{i} + 7\underline{j}$ and is moving with speed 15 ms^{-1} in the direction $3\underline{i} - 4\underline{j}$. Find its position vector after 2s.

unit vector $= \frac{3\underline{i} - 4\underline{j}}{\sqrt{3^2+(-4)^2}} = \frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}$ ⇐

velocity $= 15 \times \left(\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right)$

$= 9\underline{i} - 12\underline{j}$

$r = r_0 + vt$

$r = 4\underline{i} + 7\underline{j} + 2(9\underline{i} - 12\underline{j})$

$r = 4\underline{i} + 7\underline{j} + 18\underline{i} - 24\underline{j}$

$r = 22\underline{i} - 17\underline{j}$ ⇐

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Force and acceleration :-

$a = \frac{v-u}{t}$ and $v = u + at$

$F = ma$

worked example ⑧ :

A particle P has velocity $(-3\underline{i} + \underline{j})$ at $t=0$. The particle moves with constant acceleration $a = (2\underline{i} + 3\underline{j})\,ms^{-2}$. Find the speed of the particle after 3 seconds.

$u = -3\underline{i} + \underline{j}$ $\quad v = u + at$

$a = 2\underline{i} + 3\underline{j}$ $\quad v = (-3\underline{i} + \underline{j}) + 3(2\underline{i} + 3\underline{j})$

$t = 3$ $\quad v = -3\underline{i} + \underline{j} + 6\underline{i} + 9\underline{j}$

$v = 3\underline{i} + 10\underline{j}$

$sp = \sqrt{3^2 + 10^2}$

$= 10.4\ ms^{-1}$ ⇐

worked example ⑨ 8

A constant force F N acts on a particle of mass 2 kg. For 10s. The particle is initially at rest and 10s later it has a velocity of $(10\underline{i} - 24\underline{j})\,ms^{-1}$ find F.

$a = \frac{v-u}{t} = \frac{10\underline{i} - 24\underline{j}}{10} = \underline{i} - 2.4\underline{j}$

$F = 2(\underline{i} - 2.4\underline{j})$

$F = 2\underline{i} - 4.8\underline{j}$ ⇐

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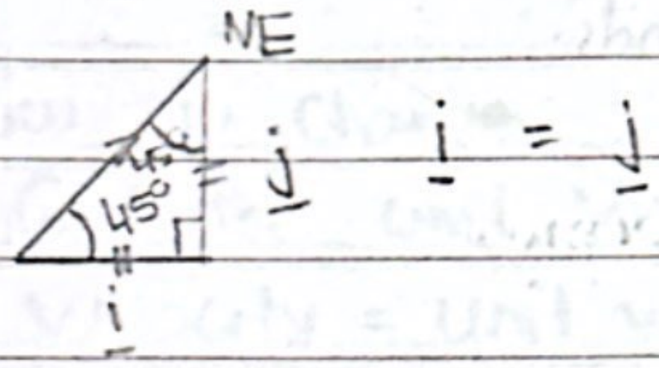
Facts :-

Position vector B = $\overrightarrow{OB}$ where O is the origin

$\overrightarrow{AB}$ = Finding point - starting point

= $\overrightarrow{OB}$ - $\overrightarrow{OA}$

1- When something moves in any of the 4 sub-directions, [NE, SE, NW, SW] it forms a 90° isosceles triangle, vertical component $\underline{j}$ = Horizontal component $\underline{i}$



2- Particle B moves due east / west of particle A

$\underline{j}$ component = 0 ; since there is no vertical movement

3- Particle M moves due North/south of particle L

$\underline{i}$ component = 0, since there is no Horizontal movement

4- Ship A is in a subdirection of ship B. Given position vector A and B.

r_A, r_B

① Get a vector from B to A or A to B (depending on the Qn) $\overrightarrow{AB} = r_B - r_A$

② The $\underline{i}$ and $\underline{j}$ components of $\overrightarrow{AB}$ are equal, since the 2 ships are in a subdirection relative to each other.

5- Whenever you are asked to find minimum / maximum distance use your differentiation.

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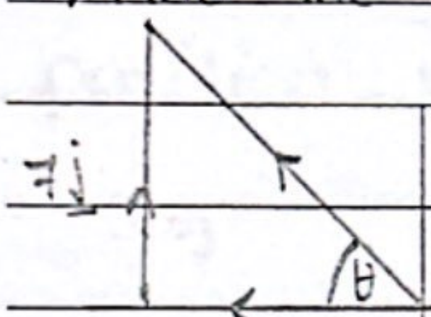
Past Paper Qns 8-

Jan 2019 Q2

1- At time $t=0$, a bird A leaves its nest, that is located at the point with position vector $(20\underline{i} - 17\underline{j})$ m and flies with constant velocity $(-6\underline{i} + 7\underline{j})$ ms^{-1}. At the same second a second Bird B leaves its nest at position vector $(-8\underline{i} + 9\underline{j})$ m and flies with constant velocity $(P\underline{i} + 2P\underline{j})$ ms^{-1}. where P is a constant. At time $t=4$ bird B is southwest of bird A.

a) Find the direction of motion A as a bearing.



$\theta = \tan^{-1}(7/6) = 49.4°$

Bearing $= 270 + 49.4$

$= 319°$ ⇐

b) Find the speed of B

$\underline{r}_A = 20\underline{i} - 17\underline{j} + 4(-6\underline{i} + 7\underline{j})$

$= -4\underline{i} + 11\underline{j}$ ← ①

$\underline{r}_B = -8\underline{i} + 9\underline{j} + 4(P\underline{i} + 2P\underline{j})$

$= (4P-8)\underline{i} + (9+8P)\underline{j}$ ←

Plan :-

① Get position vectors of A and B

② subtract them

③ equate $\underline{i}$ and $\underline{j}$ components

④ solve for P

⑤ |velocity| = speed

$\underline{r}_A - \underline{r}_B$ ②

$-4\underline{i} + 11\underline{j} - [(4P-8)\underline{i} + (9+8P)\underline{j}]$

$= (-4 - 4P + 8)\underline{i} + (11 - 9 - 8P)\underline{j}$

$= (4-4P)\underline{i} + (2-8P)\underline{j}$ ←

③, ④

$4 - 4P = 2 - 8P$

$P = -\frac{1}{2}$ ←

⑤ velocity $= (-\frac{1}{2}\underline{i} - \underline{j})$ ms^{-1}

$|\text{velocity}| = \sqrt{(-\frac{1}{2})^2 + (-1)^2}$

$= 1.12$ ms^{-1} ⇐

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Oct 2018 Q6

2- The point A on a Horizontal playground has position vector $(3\underline{i} - 2\underline{j})$m. At time $t=0$, a girl kicks a ball from A. The ball moves horizontally along the playground with constant velocity $(4\underline{i} + 5\underline{j})ms^{-1}$. Modelling the ball as a particle, find:

a) The speed of the ball

$|v| = \sqrt{4^2 + 5^2}$

$= 6.40\ ms^{-1}$ ⇐

b) The position vector of the ball at time t seconds

$r_{ball} = r_0 + vt$

$= 3\underline{i} - 2\underline{j} + t(4\underline{i} + 5\underline{j})$

$= (3+4t)\underline{i} + (5t-2)\underline{j}$ ⇐

The point B on the playground has position vector $(\underline{i} + 6\underline{j})$m. At time = T seconds the ball is due east of B.

c) Find the value of T

x B - - - - ⊗ ball

* Notice that both, ball and point B are on the same level, Hence have equal $\underline{j}$ components.

J-component Ball = 6

$r_{Ball} = (3+4T)\underline{i} + (5T-2)\underline{j}$

$5T - 2 = 6$

$T = 8/5$

$T = 1.6\ s$ ⇐

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A Boy running due east with constant speed $v ms^{-1}$. At the instant when the girl kicks the ball from A, the boy is at B.

Given that the boy intercepts the ball

$r_{Boy} = r_0 + vt$

due east
$j = 0$

$v(i + 0)$

$r_0 = i + 6j$

$v = vi + 0$

$t = t$

$r_{Boy} = i + 6j + t(vi)$

$= vi + v0$

$= vi$

$r_{Boy} = (1 + vt)i + 6j$

* Since Boy and Ball intercept therefore, their final position vectors are equal !!

$r_{Boy} = r_{Ball}$

$(1 + vt)i + 6j = (3 + 4t)i + (5t - 2)j$

$6 = 5t - 2$ → equating j-components

$t = 1 \cdot 6$

$1 + vt = 3 + 4t$ → equating i-components

$1 + v(1 \cdot 6) = 3 + 4(1 \cdot 6)$

$1 \cdot 6v + 1 = 3 + 6 4$

$1 \cdot 6v = 8 \cdot 4$

$v = 5 \cdot 25$ ⟸

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Jan 2021 / /

3- Two girls, Agatha and Brionie, are roller skating in a building. They are modelled as particles.
At time $t=0$ Agatha is at the point with position vector $(11\underline{i} + 11\underline{j})$m and Brionie is at point with position vector $(7\underline{i} + 16\underline{j})$m.
Agatha skates with velocity $(3\underline{i} - \underline{j})$ ms^{-1}
Brionie skates with velocity $(4\underline{i} - 2\underline{j})$ ms^{-1}

a) Find the position vector of Agatha at time = ts

$r = r_0 + vt$

$r = (11\underline{i} + 11\underline{j}) + (3\underline{i} - \underline{j})t$

$= (11 + 3t)\underline{i} + (11 - t)\underline{j}$ ⟸

• At time = b s Agatha passes through point P

b) show that Berionie also passes through P and find t when this occurs.

Position vector Agatha at $t = 6$ = Positin vector Brionie at $t = t$

$r_{A \text{ at } t=6} = (11 + (3 \times 6))\underline{i} + (11 - 6)\underline{j}$

$= 29\underline{i} + 5\underline{j}$ ←

$r_{B \text{ at } t=t} = (7\underline{i} + 16\underline{j}) + (4\underline{i} - 2\underline{j})t$

$= (7 + 4t)\underline{i} + (16 - 2t)\underline{j}$ ←

You can equate either $\underline{i}$ or $\underline{j}$ components to find t.

$5 = 16 - 2t$ → equating $\underline{j}$ components

$2t = 11$

$t = 5.5$ s ⟸

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At time t seconds, Agatha is at point A and Brionie is at B.

c) show that $\overrightarrow{AB} = [(t-4)\underline{i} + (5-t)\underline{j}]$ m

$\underline{r}_B - \underline{r}_A$

$(7+4t)\underline{i} + (16-2t)\underline{j} - (11+3t)\underline{i} - (11+t)\underline{j}$

$(7+4t-11-3t)\underline{i} + (16-2t-11+t)\underline{j}$

$= (t-4)\underline{i} + (5-t)\underline{j}$ ⇐ As required

d) Find the distance between the two girls when they are closest together.

The Qn implies that it wants us to find the MINIMUM DISTANCE.

Distance $= |\overrightarrow{AB}|$

$d = \sqrt{(t-4)^2 + (5-t)^2}$

$d^2 = (t-4)^2 + (5-t)^2$

$d^2 = t^2 - 8t + 16 + 25 - 10t + t^2$

$d^2 = 2t^2 - 18t + 41$

METHOD 1 (calc)

$\frac{d(d^2)}{dt} = 4t - 18$

$4t - 18 = 0$ → min point

$t = 4.5$ ←

METHOD 2 (critical point)

$t = \frac{-b}{2a}$

$= \frac{-(-18)}{2 \times 2}$

$t = 4.5$ ←

$d = \sqrt{(\frac{9}{2} - 4)^2 + (5 - \frac{9}{2})^2}$

$d = 0.71$ m ⇐

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Jun 2021 Q5 / /

4 - A particle P is moving with constant acceleration. At 2pm the velocity of P is $(3\underline{i} + 5\underline{j})$ kmh^{-1} and at 2130 Pm the velocity of P is $(\underline{i} + 7\underline{j})$ kmh^{-1}
At time T hours after 2pm, P is moving in the direction of the vector $(-\underline{i} + 2\underline{j})$.

a) Find the value of T

$u = 3\underline{i} + 5\underline{j}$ $\quad a = \frac{v-u}{t}$

$v = \underline{i} + 7\underline{j}$

$t = 1/2$ $\quad a = \frac{\underline{i} + 7\underline{j} - 3\underline{i} - 5\underline{j}}{1/2}$

$a = \frac{-2\underline{i} + 2\underline{j}}{2}$ → $a = -4\underline{i} + 4\underline{j}$ ←

$v = u + at$

$v = 3\underline{i} + 5\underline{j} + (+4\underline{i} + 4\underline{j})T$ → $6 - 8T = -5 - 4T$

$= (3 - 4T)\underline{i} + (5 + 4T)\underline{j}$ $\quad 11 = 4T$

Using parallilism $\quad T = 2.75$ ⇐

$\frac{3 - 4T}{5 + 4T} = \frac{-1}{2}$

Another particle Q has velocity v_Q kmh^{-1} at time t hours after 2pm where $v_Q = (-4 - 2t)\underline{i} + (k + 3t)\underline{j}$ where k is a constant. Given that there is an instant when the velocity of P = Q

b) Find the value of k

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$V_P = (3-4t)\underline{i} + (5+4t)\underline{j}$

$V_Q = (-4-2t)\underline{i} + (M+3t)\underline{j}$

V_P $\underline{i}$-component $=$ V_Q $\underline{i}$-component

$3-4t = -4-2t$

$7 = 2t$

$t = 3.5 \leftarrow$

V_P - $\underline{j}$ component $=$ V_Q $\underline{j}$ component

$5+4t = M+3t$ → Sub. $t = 3.5$

$5+4(3.5) = M+3(3.5)$

$19 = M + 10.5$

$M = 8.5 \Leftarrow$

Jan 2023 Q3-

5 - A Particle P is moving with constant acc $(-4\underline{i} + \underline{j})ms^{-2}$

At time $t=0$, P has velocity $(14\underline{i} - 5\underline{j})ms^{-1}$

a) Find the speed of P at time $t=2$ seconds

$V = U + at$

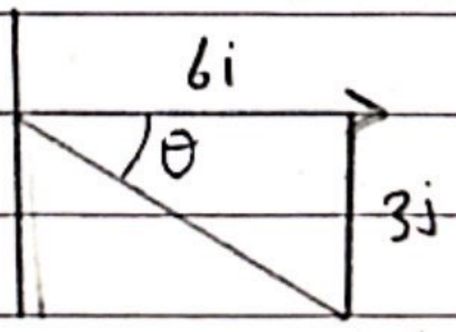
$V = 14\underline{i} - 5\underline{j} + 2(-4\underline{i} + \underline{j})$

$= 6\underline{i} - 3\underline{j}$

$|V| = \sqrt{6^2 + (-3)^2} = 6.71 \ ms^{-1} \Leftarrow$

b) the size of the angle between direction of $\underline{i}$ and the direction of motion of P at $t=2$

using V



$\theta = \tan^{-1}(3/6)$

$\theta = 27^\circ \Leftarrow$

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c) At time $t=T$, P is moving in the direction of vector $(2\underline{i}-3\underline{j})$ find T

$$V = 14\underline{i} - 5\underline{j} + (-4\underline{i} + \underline{j})t$$
$$= (14-4t)\underline{i} + (-5+t)\underline{j}$$

using parallelism

$$\frac{14-4t}{-5+t} = \frac{2}{-3}$$
$$-42+12t = 2t-10$$
$$10t = 32$$
$$t = 3.2 \Leftarrow$$

Jan 2005 Q7

6- Two ships P and Q are travelling at night with constant velocities. At midnight, P is at the point with position vector $(20\underline{i}+10\underline{j})$ km realtive to a fixed origin O. At the same time Q is at the point with position vector $(14\underline{i}-15\underline{j})$ Km. Three hours later. P is at the point with position vector $(29\underline{i}+34\underline{j})$ Km. The ship Q travels with velocity $12\underline{j}$ Kmh^{-1}. At time t hours after midnight, the position vectors of P and Q are p Km and q Km. Find:

a) The velocity of P in terms of i and j

$$r = r_0 + vt$$
$$29\underline{i} + 34\underline{j} = 20\underline{i} + 10\underline{j} + 3v$$
$$\frac{9\underline{i} + 24\underline{j}}{3} = v$$
$$v = 3\underline{i} + 8\underline{j}$$

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b) Expressions for P and Q in terms of t, $\underline{i}$ and $\underline{j}$

$r_P = P = r_0 + vt$

$= 20\underline{i} + 10\underline{j} + (3\underline{i} + 8\underline{j})t$

$P = (20 + 3t)\underline{i} + (10 + 8t)\underline{j}$ ⇐

$r_Q = Q = 14\underline{i} - 6\underline{j} + (12\underline{j})t$

$= 14\underline{i} + (12t - 6)\underline{j}$ ⇐

c) At time t hours after midnight, the distance PQ = d km
By finding an expression for $\overrightarrow{PQ}$ show that $d^2 = 25t^2 - 92t + 292$

$\overrightarrow{PQ} = q - p$

$= 14\underline{i} + (12t - 6)\underline{j} - (20 + 3t)\underline{i} - (10 + 8t)\underline{j}$

$= (14 - 20 - 3t)\underline{i} + (12t - 6 - 10 - 8t)\underline{j}$

$= (-6 - 3t)\underline{i} + (4t - 16)\underline{j}$

$|\overrightarrow{PQ}| = \sqrt{(-6 - 3t)^2 + (4t - 16)^2} = d$

$d^2 = (-6 - 3t)^2 + (4t - 16)^2$

$= 36 + 36t + 9t^2 + 16t^2 - 128t + 256$

$= 25t^2 - 92t + 292$ ⇐ As required

Weather conditions are such that an observer on P can only see the lights on Q when distance between P and Q is 15 Km or less. Given that when t=1, the lights on Q move into the sight of the observer,

d) Find the time to the nearest minute at which the lights on Q move out of sight of the observer

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$d = 15$

$15^2 = 25t^2 - 92t + 292$

$225 = 25t^2 - 92t + 292$

$25t^2 - 92t + 67 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = 1$, $t = 2.68$

time $= 2.68$ hrs

2.68×60

$= 161$ mins $\Leftarrow$

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